

## Vectors as Relative Positions:

### A Compositional Semantics of Modified PPs<sup>1</sup>

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#### *Summary*

The interpretation of modified PPs, like *one meter behind the desk*, *far outside the village*, or *right under the lamp* has never received much attention in the literature about prepositions. However, as this paper shows, the modification of PPs presents an intriguing problem for a compositional semantics of PPs. This problem can be solved when the PP is interpreted, not as a set of points or mereological portions of space, but as a set of *vectors*, that represent positions relative to the reference object. Modifiers map a set of vectors to a subset. For example, if *behind the desk* denotes the set of vectors pointing backward from the desk, then *one meter behind the desk* denotes the subset consisting of those vectors that have a length of one meter. Given the familiar operations on vectors, the denotations of PPs can be studied in a systematic way, by formulating formal properties that characterize empirically relevant subclasses of locative PPs or that provide general constraints on their denotation. The examples are taken from Dutch, but the conclusions are valid for all languages that have the kind of PP modification that exists in Dutch and English.

Although there is a wealth of literature about many aspects of the interpretation of prepositions and PPs, little attention has been paid to the semantics of *modified* PPs, like those in (1):

- (1) a. one meter behind the desk
- b. far outside the village
- c. right under the lamp

The discussion of cases like these is mainly limited to some accidental remarks in the semantic literature, e.g. Cresswell (1978:14, 29-31), Jackendoff (1983:167), Hawkins (1984:330-336), Bierwisch (1988:32, 44-45, 48-49), Lang (1991:133) and Wunderlich & Herweg (1991:780). Only Wunderlich & Kaufmann (1990) and Rauh (1996) discuss the semantics of modified PPs in more depth.

The purpose of this paper is not to give a complete and detailed treatment of modified PPs that would fill this gap or to discuss and compare the proposals in the literature just mentioned, but rather to demonstrate, by focussing on certain modifiers of locative PPs in Dutch, that PP modification presents an intriguing problem for any compositional semantics of prepositions and PPs and that the most natural solution to this problem involves the use of *vectors*, interpreting the PP as a particular set of vectors emanating from the reference object. However, the use of vectors has implications that go far beyond the domain of modification, because the notion of vector allows us to study the denotation of PPs (whether modified or not) in a systematic and precise way, making use of familiar algebraic operations on vectors. In this way natural classes of prepositions and general constraints on prepositional meanings can be formulated.

After the introduction in section 1 of the basic problem, section 2 presents the outlines of a solution that is based on vectors. Section 3 provides some mathematical background for the semantic definitions of individual locative prepositions and modifiers in section 4 and for the formulation in section 5 of a range of relevant formal properties of PP denotations.

## 1 Modification of PPs

The empirical scope of this paper is restricted to the the most important locative prepositions of Dutch and the most common modifiers. The locative prepositions that will be discussed in this paper are given in (2):

- (2) voor (in front of), achter (behind)  
boven (above, over), onder (below, under)  
binnen (inside), buiten (outside)  
links van (left of), rechts van (right of)  
naast (next to, beside), tussen (between)  
in (in), op (on, at), bij (near)

For the purposes of this paper, locative prepositions are to be understood as those prepositions which, in combination with a noun phrase object, are primarily used to denote locations, typically as the complement of the copula *be*. Directional prepositions like *naar* (to), *van* (from), *uit* (from), *door* (through), and *tegen* (against) will be discussed only very briefly, because they do not denote locations, but *paths*, which I will analyze as sequences of locations. Prepositions like *langs* (along), *over* (over), *om* (around) and *voorbij* (past, beyond) may be used locatively, either by distributing a plural or elongated object along a path (as in (3a)), or by identifying a location as the endpoint of a path (in (3b)):

- (3) a. de bomen / het hek om het huis  
the trees / the fence around the house  
b. Het hotel is over de heuvel / om de hoek  
The hotel is over the hill / around the corner

However, I will follow the common assumption in the literature that these prepositions primarily refer to paths and that their locative use is derived.<sup>2</sup> Several locative prepositions, like *aan* ('on' in the meaning of 'attached to'), *halverwege* (halfway between), and *tegenover* (opposite) are absent from the list in (2) because their analysis involves complications that would go beyond the basic proposal of this paper.

PPs formed with the prepositions in (2) can be modified in different ways, two of which will be discussed in this paper: modification of *distance* and modification of *direction*.

The *distance modifier* can be a measure phrase:

- (4) a. twee centimeter boven de deur  
two centimeters above the door
- b. enkele stappen achter het doel  
a few steps behind the goal
- c. vele lichtjaren buiten de melkweg  
many lightyears outside the milky way

Some of the so-called dimensional adjectives, namely *hoog* (high), *laag* (low), *diep* (deep), *ver* (far), and *dicht* (close) can also specify a distance in a particular direction:

- (5) a. hoog boven de deur  
high above the door
- b. diep onder de grond  
deep under the ground

- c. dicht achter de gouden koets  
close behind the golden coach

A group of special adverbs indicates a very short distance, close to zero:

- (6) a. direct boven de deur  
directly above the door
- b. net buiten de stad  
just outside the city
- c. vlak naast me  
right beside me

The *direction modifiers* specify the direction of the location, either by making reference to the angle that the direction makes with respect to an axis, like *recht* and *pal* (straight) and *schuin* (diagonally):

- (7) a. recht boven de deur  
straight above the door
- b. schuin achter de tafel  
diagonally behind the table

or by means of the adverbs *links* (left) and *rechts* (right), yielding phrases that have no straightforward translation in English:

- (8) a. links boven de deur

above the door and to the left of it

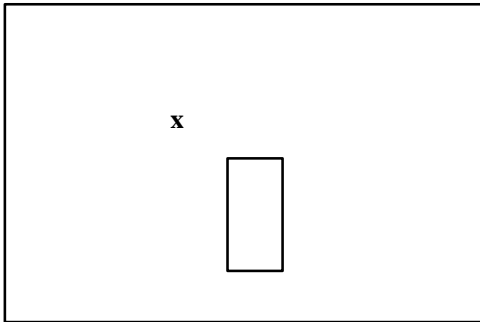
b. rechts voor het doel

in front of the goal and to the right of it

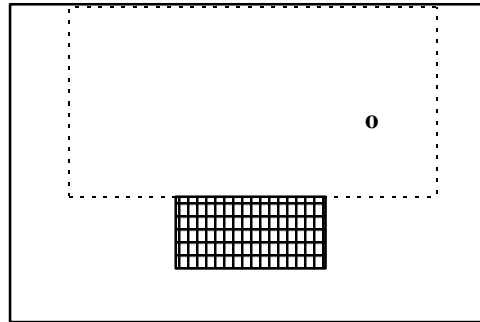
In order to make clear what these PPs mean, the following pictures will be helpful:

(9)

a. *links boven de deur*



b. *rechts voor het doel*



In (9a) the nail x is *links boven de deur* and in (9b) the ball o is *rechts voor het doel* (when seen from above).

How do these modified PPs fit into the semantic analysis of prepositions and PPs? Traditionally, a locative preposition like *boven* (above) is taken to be a relation between a *theme* x (also called trajector or figure) and a *reference object* y (also called landmark or ground):

(10) BOVEN ( x , y )

However, in much recent work this relation is broken down into two parts, as in (11):

(11)  $\text{LOC} ( x , \text{BOVEN} ( y ))$

The function BOVEN maps the reference object to a *region* or *place* BOVEN(y) and the theme is located in this region by a general location relation.<sup>3</sup> I will assume in this paper that this location relation LOC is actually not part of the lexical meaning of *boven* or any other preposition, but contributed by the predicative and modificational constructions in which a locative PP is used (e.g. *de spijker (is) boven de deur* ‘the nail (is) above the door’). The preposition *boven* only contributes the function BOVEN; this means that the theme x is not part of the lexical meaning of the preposition, but provided by a ‘type-shift’ or ‘coercion’ operation that turns locative PPs into predicates. As a result the basic denotation of a locative PP like *boven de deur* (above the door), corresponding to the region part BOVEN(y) in (11), is a purely spatial object. Modifiers of PP apply to this region before the location operation maps the region to a set of objects, as in *recht boven de deur* (straight above the door):

(12)  $\text{LOC} ( x , \text{RECHT} ( \text{BOVEN} ( y )))$

Suppose now for the sake of concreteness that the region corresponding to *boven de deur* is a set of points, which is the natural mathematical way to model a spatial region. Then the modifiers of this PP in the a-examples of (4) to (8) each have to be interpreted as functions that map a set of points to a set of points, presumably a subset.<sup>4</sup> These functions could provisionally be formulated in the following way, with  $\llbracket \alpha \rrbracket$  representing the denotation of an expression  $\alpha$ :

(13) a.  $\llbracket 2 \text{ cm PP} \rrbracket = \{ p \in \llbracket \text{PP} \rrbracket \mid p \text{ is 2 centimeters} \}$   
b.  $\llbracket \text{hoog PP} \rrbracket = \{ p \in \llbracket \text{PP} \rrbracket \mid p \text{ is high} \}$

- c.  $\llbracket \text{direct PP} \rrbracket = \{ p \in \llbracket \text{PP} \rrbracket \mid p \text{ is direct} \}$
- d.  $\llbracket \text{recht PP} \rrbracket = \{ p \in \llbracket \text{PP} \rrbracket \mid p \text{ is straight} \}$
- e.  $\llbracket \text{links PP} \rrbracket = \{ p \in \llbracket \text{PP} \rrbracket \mid p \text{ is to the left} \}$

However, a moment's reflection makes clear that these definitions do not make sense, because all by itself a point cannot be said to be 'two centimeters', or 'direct', or 'straight'. The definitions in (13b) and (13e) make more sense at first sight, but even in those cases a point can only be 'high' and 'to the left' with respect to a reference point. In *hoog boven de deur* (high above the door) and *links boven de deur* (above and to the left of the door), *hoog* and *links* take the door as their reference point and not the ground or the speaker (which are the usual reference points for *hoog* and *links*). *Relativity* is the general characteristic of the modifiers in (13): they do not specify absolute properties of the points of the region, but they specify the *distance* between the point and the reference object (in (13a), (13b), and (13c)) or the *direction* of the point with respect to the reference object (in (13d) and (13e)). This relational character can be accounted for in the following way:

- (14) a.  $\llbracket \text{2 cm PP} \rrbracket = \{ p \in \llbracket \text{PP} \rrbracket \mid 2\text{cm}(p, \llbracket \text{NP} \rrbracket) \}$
- b.  $\llbracket \text{hoog PP} \rrbracket = \{ p \in \llbracket \text{PP} \rrbracket \mid \text{high}(p, \llbracket \text{NP} \rrbracket) \}$
- c.  $\llbracket \text{direct PP} \rrbracket = \{ p \in \llbracket \text{PP} \rrbracket \mid \text{direct}(p, \llbracket \text{NP} \rrbracket) \}$
- d.  $\llbracket \text{recht PP} \rrbracket = \{ p \in \llbracket \text{PP} \rrbracket \mid \text{straight}(p, \llbracket \text{NP} \rrbracket) \}$
- e.  $\llbracket \text{links PP} \rrbracket = \{ p \in \llbracket \text{PP} \rrbracket \mid \text{left}(p, \llbracket \text{NP} \rrbracket) \}$

Of course, the relation that defines a modifier would have to be worked out in order to capture its full lexical meaning. For example, *direct* requires that the distance between *p* and the



reference object is almost zero and for *recht* the angle that the line between p and the reference object makes with the relevant axis is zero. I will come back to the specification of the lexical semantics of modifiers in section 4.2, but here the schematic interpretations in (14) suffice.

The definitions in (14) make sense, but now another problem arises. The definitions in (14) are actually non-compositional: the interpretation of a modified PP is not a function of its immediate constituents (the modifier and the inner PP), but the proper interpretation of PP modification requires access to the reference object NP, which is strictly speaking not visible to the interpretation process. When locative PPs are taken to denote spatial regions then a compositional interpretation of modified PPs becomes impossible.

One possible way out worth considering is that modifiers are not sisters of PP (or P-bar), but of the preposition:

(15) [ [ ver [P boven] ] [NP de deur ] ]

If this would be the case, then modifiers map relations to relations in a way that is both compositional and descriptively adequate:

- (16) a.  $[[2\text{ cm } P]] = \{ (x,y) \in [[P]] \mid 2\text{cm}(x,y) \}$   
 b.  $[[\text{hoog } P]] = \{ (x,y) \in [[P]] \mid \text{high}(x,y) \}$   
 c.  $[[\text{direct } P]] = \{ (x,y) \in [[P]] \mid \text{direct}(x,y) \}$   
 d.  $[[\text{recht } P]] = \{ (x,y) \in [[P]] \mid \text{straight}(x,y) \}$   
 e.  $[[\text{links } P]] = \{ (x,y) \in [[P]] \mid \text{left}(x,y) \}$

However, it is generally assumed in the syntactic literature that modifiers inside PP are not sisters of the preposition, but of the combination of preposition and object, a constituent that is sometimes called P-bar (e.g. Jackendoff 1973, Van Riemsdijk 1978), and we would want to have good syntactic reasons to depart from this assumption, rather than just to get our semantics right. Furthermore, Dutch shows that modification of prepositions can never be a general solution to the problem. In Dutch, the reference object can always be replaced by a special proform that can come between a modifier and the preposition:<sup>5</sup>

- (17) a. twee centimeter er boven  
two centimeters above it
- b. hoog daar boven  
high above that
- c. direct hier boven  
directly above this
- d. recht waar boven  
straight above which
- e. links ergens boven  
above and to the left of something

In these examples the modifier can not be a functor applying to the preposition, mapping a relation into a relation. It must be the PP (or P-bar) that is modified.

Concluding, if we want to interpret modifiers compositionally as functions applying to the denotation of a PP, then a denotation based on points is simply not adequate. The heart of the problem is that modifiers of PP do not refer to positions denoted by the locative PP, but to *distances* and *directions* relative to the reference object. PP modification can only be

compositional if these aspects are somehow directly reflected in the denotation of a locative PP.

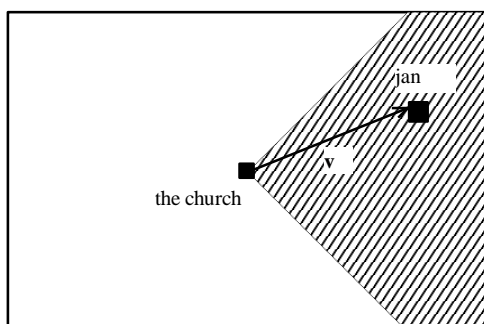
## 2 Vectors as relative positions

The proposal of this paper is to analyze a region as a set of *vectors*. The region denoted by the PP *achter de kerk* (behind the church), for example, will then be the set of vectors with their starting point at the church that point backwards and the theme of this PP is located at the end point of one of these vectors. The truth conditions of (18a) are as in (18b), with  $\mathbf{v}$  a variable over vectors:

- (18) a. Jan is achter de kerk  
Jan is behind the church
- b.  $\exists \mathbf{v} [ \mathbf{v} \in [\text{achter de kerk}] \wedge \text{loc}(\text{jan}, \mathbf{v}) ]$

How to define the region in (18b) will be explained in section 4.1, but the picture in (19) gives a good impression, with the shaded area representing the region behind the church and  $\mathbf{v}$  being one of the vectors from this region:<sup>6</sup>

- (19) *Jan is achter de kerk*



Other prepositions can be analyzed vector-theoretically along the same lines:

- (20) a. *bij de kerk* (near the church): the set of vectors with their origin at the church with a length smaller than a pragmatically determined number  $r$
- b. *boven de kerk* (above the church): the set of vectors that point from the church in an upward direction
- c. *tussen de kerk en de kroeg* (between the church and the pub): the vectors that point from the church towards the pub and vice versa

The general idea should be clear at an intuitive level. What the vector does is formalize the notion of a *relative position*, i.e. a position specified in relation to a reference object that functions as a spatial origin. By its very nature the vector concept provides the parameters of *distance* and *direction* that prepositions use to specify relative positions and that can be further specified by modifiers.

Using vectors, the general semantics of the modifiers is straightforward. Each modifier selects from the set of vectors in the extension of the PP those vectors that have a particular length and/or direction:

- (21) a.  $\llbracket 2 \text{ cm PP } \rrbracket = \{ \mathbf{v} \in \llbracket \text{PP} \rrbracket \mid 2\text{cm}(\mathbf{v}) \}$
- b.  $\llbracket \text{hoog PP} \rrbracket = \{ \mathbf{v} \in \llbracket \text{PP} \rrbracket \mid \text{high}(\mathbf{v}) \}$
- c.  $\llbracket \text{direct PP} \rrbracket = \{ \mathbf{v} \in \llbracket \text{PP} \rrbracket \mid \text{direct}(\mathbf{v}) \}$
- d.  $\llbracket \text{recht PP} \rrbracket = \{ \mathbf{v} \in \llbracket \text{PP} \rrbracket \mid \text{straight}(\mathbf{v}) \}$
- e.  $\llbracket \text{links PP} \rrbracket = \{ \mathbf{v} \in \llbracket \text{PP} \rrbracket \mid \text{left}(\mathbf{v}) \}$

The measure phrase *2 cm* selects those vectors from the PP denotation that have a length of two centimeters and the adverb *recht* (straight) picks out those vectors that point in a particular direction with respect to a reference axis. The exact lexical definition of these and the other modifiers is not important at this moment (see section 4.2 for this). What is crucial is that modification of PPs can be done in a compositional and natural way, because spatial entities are used that are by their very nature relational and that carry information about the reference object that would otherwise have only been accessible to the modifier in a non-compositional way.

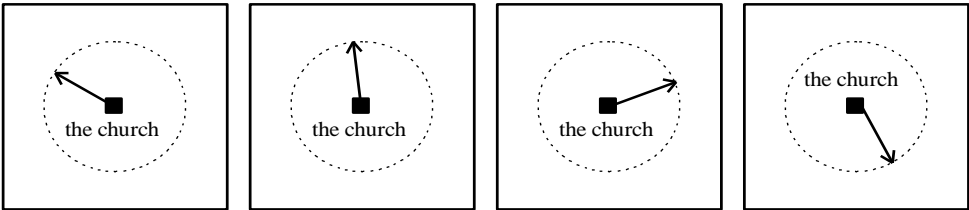
Let me briefly compare this proposal to two alternative approaches to the modification problem. The notion of relative position might be modelled more directly, by adding the reference object of a PP to each position *p* in the region, yielding a set of pairs of a position *p* and the reference object *y*.<sup>7</sup> A modifier like *2 cm* would then single out those pairs (*p,y*) such that the distance between *p* and *y* is two meters. This seems to yield the same result as a vector-based approach, but within a more traditional point-based framework. Another possibility is explored by Wunderlich & Kaufmann (1990) and Wunderlich & Herweg (1991) within the ‘two-level approach’ that separates a compositional semantic level from a non-compositional conceptual level.<sup>8</sup> In the example *hoog boven de deur* (high above the door), the modifier *hoog* is a two-place predicate *high(p,c)* expressing the vertical distance between a point *p* and a reference point *c*. At the semantic level this variable *c* is left unspecified, but at the conceptual level it is identified with the reference object of the preposition in a non-compositional way.

However, both alternatives solve the problem without gaining any extra insights into the nature of locative PPs and their modifiers. First, PP modifiers like *recht* (straight) and *schuin* (diagonally), that are also used to orient linear objects (e.g. *een rechte/schuine streep* ‘a straight/diagonal line’) suggest that the relation expressed by a locative preposition behaves in

this respect like a *line segment*, which is something we get with vectors but not with pairs of a point and a reference object or with modifiers that have an implicit reference point that needs to be specified conceptually. Second, a denotation based on vectors provides the algebraic structure that reveals the kind of properties discussed in section 5, properties that can only be formulated in an indirect way without vectors.

Although vectors are intuitively connected to movement and in physics used to represent change of position, here their function is exclusively restricted to represent position. The path or movement of a directional PP like *naar de kerk* (to the church), for example, is not treated as a vector pointing to the church, but as a *sequence* of positions, the last one of which coincides with the church (see for instance Langacker 1986, Habel 1989, Nam 1995). Given our proposal to treat positions as vectors, this means that a path leading to the church is a sequence of vectors, each having their starting point at the church and gradually going to zero length. There is a good reason why a path cannot be represented by one single vector: because the paths corresponding to some prepositions are not linear. For example, the PP *om de kerk* (around the church), denotes a set of paths that are approximately circles enclosing the church. Clearly, a vector cannot be used to represent such a circular path, but we can represent a path around the church as a sequence of vectors. The following diagram gives a very rough idea of what such a path would look like:

(22) *om de kerk*



This much will have to suffice to indicate how a semantics of directional prepositions can be built on a theory with vectors. In the remainder of this paper we will deal exclusively with locative prepositions.

### 3 The algebra of vectors

We have seen that the PP denotes a set of vectors that all have their origin in the reference object of the PP. Mathematically, the set total of all vectors with the same origin corresponds to the algebraic notion of a *vector space*:

(23) A vector space  $\mathbf{V}$  over the set of real numbers  $\mathbf{R}$  is a set that is closed under two operations:

a. *Vector addition*

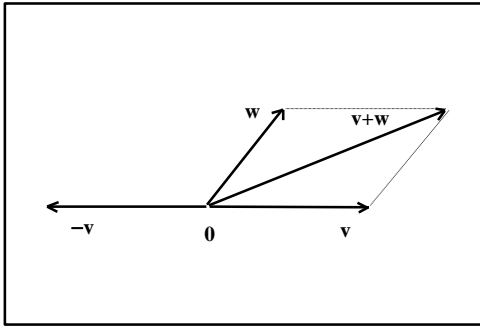
For every pair  $\mathbf{v}, \mathbf{w} \in \mathbf{V}$  there is exactly one  $\mathbf{v} + \mathbf{w} \in \mathbf{V}$ , the *vector sum* of  $\mathbf{v}$  and  $\mathbf{w}$

b. *Scalar multiplication*

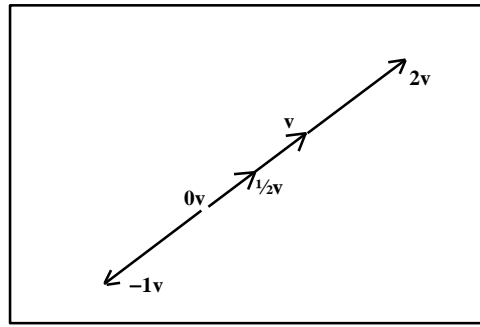
For every  $\mathbf{v} \in \mathbf{V}$  and  $s \in \mathbf{R}$  there is exactly one  $s\mathbf{v} \in \mathbf{V}$ , the *scalar product* of  $\mathbf{v}$  by scalar  $s$

The operations of vector addition and scalar multiplication are graphically illustrated in (24) and (25), respectively:

(24) *Vector addition*



(25) *Scalar Multiplication*



A vector space has the following properties:

- (26) a. For all  $\mathbf{u}$  and  $\mathbf{v} \in \mathbf{V}$ ,  $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
- b. For all  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w} \in \mathbf{V}$ ,  $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
- c. There is an element  $\mathbf{0} \in \mathbf{V}$ , the *zero vector*, such that  $\mathbf{v} + \mathbf{0} = \mathbf{0} + \mathbf{v} = \mathbf{v}$   
for all  $\mathbf{v} \in \mathbf{V}$
- d. For every  $\mathbf{v} \in \mathbf{V}$  there is a  $-\mathbf{v} \in \mathbf{V}$ , the *inverse* of  $\mathbf{v}$ , such that  $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$
- e. For all  $\mathbf{u}$  and  $\mathbf{v} \in \mathbf{V}$  and every  $c \in \mathbf{R}$ ,  $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
- f. For every  $\mathbf{v} \in \mathbf{V}$  and  $a$  and  $b \in \mathbf{R}$ ,  $(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}$  and  $(ab)\mathbf{v} = a(b\mathbf{v})$
- g. For every  $\mathbf{v} \in \mathbf{V}$ ,  $1\mathbf{v} = \mathbf{v}$

In order to use vectors for the model-theoretic interpretation of natural language expressions, vectors have to be added to the standard domain  $E$  of objects. One vector space  $\mathbf{V}$  is not sufficient, however; the model will have to contain a much larger set  $\mathbf{S}$  of vectors, providing for each pair of points  $P$  and  $Q$ , a vector pointing from  $P$  to  $Q$  and a vector pointing from  $Q$  to  $P$ .  $\mathbf{S}$  is then the union of an infinite set of vector spaces, one for each point in space and this  $\mathbf{S}$  is added to the traditional domain  $E$  of individual objects.<sup>9</sup>

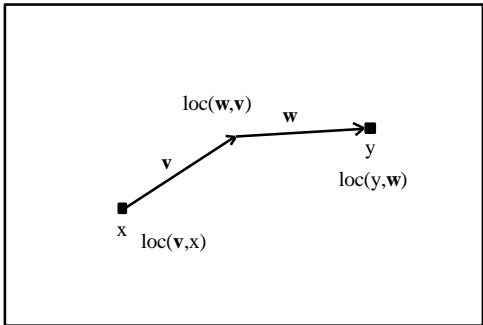


In addition to  $E$  and  $S$  (with its algebraic structure), a general location relation  $loc$  is assumed that determines the spatial relationships between  $E$  and  $S$  and between vectors of different vector spaces.  $Loc$  is a subset of the set of pairs  $(E \cup S) \times (E \cup S)$  that can be understood in the following way:

- (27) a.  $loc(x, y)$   $x = y$
- b.  $loc(v, x)$  the beginning point of vector  $v$  is located at object  $x$
- c.  $loc(y, w)$  object  $y$  is located at the end point of vector  $w$
- d.  $loc(w, v)$  the beginning point of vector  $w$  is located at the endpoint of vector  $v$

The diagram gives a rough idea of the nature of this relation:

(28) *Location relations*



Finally, it will be useful to assume a function  $||$  that assigns to each vector  $v$ , the *length* or *norm*  $|v|$  of that vector.

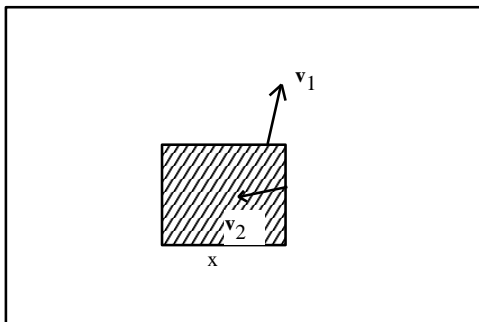
**4 Semantic definitions of prepositions and modifiers**

The interpretation of a locative prepositional phrase has the general schematic form in (29). A locative PP denotes a set of vectors taken from a ‘universe’ of vectors that is determined by the reference object NP, i.e.  $\text{space}(\llbracket \text{NP} \rrbracket_M)$ :

$$(29) \quad \llbracket [\text{PP } P \text{ NP}] \rrbracket_M = \{ \mathbf{v} \in \text{space}(\llbracket \text{NP} \rrbracket_M) \mid \dots \mathbf{v} \dots \}$$

The reference object becomes the origin of this spatial universe by selecting only those vectors that are located at the *surface* or *boundary* of the reference object, as shown in (30), and that point *outward* ( $\mathbf{v}_1$ , for *outside*, *near*, *on*, and the other prepositions) or *inward* ( $\mathbf{v}_2$ , for *in* and *inside*).

(30) *Vectors on the surface*



Evidence that location is relativized to a surface or boundary comes from examples like the following:

- (31) a. diep in de boom  
           deep in the tree  
       b. ver buiten de stad  
           far outside the city

The use of the modifier *diep* in (31a) indicates that the reference point for the position is not the tree as a whole, but its surface. When the PP *in de boom* (in the tree) denotes the set of vectors pointing from the surface of the tree inward, then *diep* can select those vectors that are long. In (31b) the modifier *ver* can only be properly understood as measuring the distance from the edge of the city. I will therefore assume that the vector universe  $\text{space}(x)$  of a reference object  $x$  consists of vectors like those in (30).<sup>10</sup> A formal definition of the function ‘space’ will not be given here, because the intuitive content is quite clear and a definition would lead us into many complexities. The preposition defines a subset of  $\text{space}(\llbracket \text{NP} \rrbracket_M)$  by imposing certain conditions on the length or orientation of vectors. For example, in the PP *boven de deur* (above the door), *boven* will take the upward vectors from the vector universe of the door:

$$(32) \quad \llbracket [\text{PP} \textit{boven} [\text{NP} \textit{de deur}]] \rrbracket_M = \{ \mathbf{v} \in \text{space}(\llbracket \textit{de deur} \rrbracket_M) \mid \text{‘upward’}(\mathbf{v}) \}$$

We already saw that generally a modifier of PP maps a set of vectors to a subset, for example:

$$(33) \quad \begin{aligned} \llbracket [\text{PP} \textit{recht} [\text{PP} \textit{boven de deur}]] \rrbracket_M &= \{ \mathbf{v} \in \llbracket \textit{boven de deur} \rrbracket_M \mid \text{‘straight’}(\mathbf{v}) \} \\ &= \{ \mathbf{v} \in \text{space}(\llbracket \textit{de deur} \rrbracket_M) \mid \text{‘upward’}(\mathbf{v}) \wedge \text{‘straight’}(\mathbf{v}) \} \end{aligned}$$

As discussed in section 1, a locative PP usually functions as a predicate, which requires that the vector-denotation must be shifted to a property of objects. This interpretation can be defined as the set of objects that are located at a vector from the region denoted by the PP:

$$(34) \quad \{ x \in E \mid \exists \mathbf{v} \in \llbracket \text{PP} \rrbracket_M \wedge \text{loc}(x, \mathbf{v}) \}$$

This is also the type of denotation that allows us to conjoin PPs, as in *achter de schuur naast de auto* (behind the barn beside the car) or *achter Jan en naast Marie* (behind Jan and beside Marie) by simply taking intersections.<sup>11</sup> There also seem to be modifiers that apply at this denotational level and not at the vector level. In the PP *hoog achter de deur* (high behind the door) *hoog* and *achter de deur* do not have the same point of reference: *hoog* does not take the door as its point of reference, but the ground. When the object level denotations of *hoog* and *achter de deur* are intersected, we get the correct result: the set of objects that are high and behind the door. Certain ambiguities with modifiers can possibly be derived from a distinction between ‘object modification’ and ‘vector modification’ of the same PP. The PP *links boven de deur* (literally, left above the door) mentioned in section 1, has another interpretation besides the one explained there and analyzed in terms of vectors in section 2: an interpretation in which *links* does not take the door as its point of reference, but a deictically given point (usually the position of the speaker). This second interpretation involves the intersection of the set of objects to the left of the speaker and the set of objects behind the door.

The focus of this paper will be exclusively on the spatial meaning of PP, under the assumption that the predicative meaning can always be derived from this spatial meaning in a systematic way, without any cost, by a ‘type-shift’ or ‘coercion’ operation (Partee 1987, Pustejovsky 1991) that maps a set of vectors to the set of objects located at those vectors, as soon as the locative PP is used in a position where a predicative expression is required. When the modified PP *recht boven de deur* is used as a predicate, the type-shift function implicit in (34) maps the set of vectors given in (33) to the following set:

$$(35) \quad \{ x \in E \mid \exists \mathbf{v} \in \llbracket \text{recht boven de deur} \rrbracket_M \wedge \text{loc}(x, \mathbf{v}) \} = \\ \{ x \in E \mid \exists \mathbf{v} \in \text{space}(\llbracket \text{de deur} \rrbracket_M) [ \text{‘upward’}(\mathbf{v}) \wedge \text{‘straight’}(\mathbf{v}) \wedge \text{loc}(x, \mathbf{v}) ] \}$$

I will now discuss the definitions of individual prepositions in section 4.1 and of modifiers in section 4.2.

#### 4.1 Definitions of prepositions

Following Herskovits (1986), Landau and Jackendoff (1993) and others, I will distinguish between the prepositions in (36) and those in (37):

(36) in (in), binnen (inside), op (on), buiten (outside), bij (near)

(37) voor (in front), achter (behind), boven (above, over), onder (below, under), links (left), rechts (right), naast (next to, besides), tussen (between)

Roughly, the prepositions in (36) express basic topological notions, like ‘inclusion’, ‘contact’, and ‘environment’. The prepositions in (37) are based on a particular direction, which is usually determined by an *axis* or by another object in the case of *tussen* (between).

Some notion of direction also plays a role within the first class of prepositions: *in/binnen* and *buiten* express opposite directions with respect to the surface of the reference object. PPs with *in* and *binnen* denote the set of vectors pointing inward and PPs with *buiten* denote the set of vectors pointing outward (all interpretations are assumed to be relative to a model *M*):

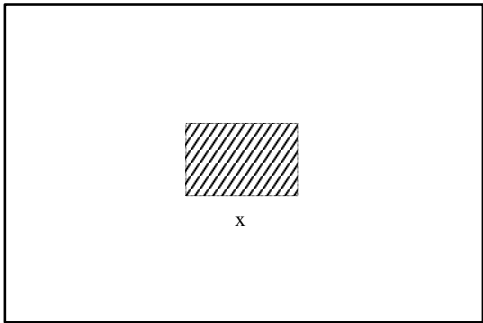
- (38) a.  $\llbracket \text{in NP} \rrbracket = \llbracket \text{binnen NP} \rrbracket = \{ \mathbf{v} \in \text{space}(\llbracket \text{NP} \rrbracket) \mid \mathbf{v} \text{ inward to } \llbracket \text{NP} \rrbracket \}$   
b.  $\llbracket \text{buiten NP} \rrbracket = \{ \mathbf{v} \in \text{space}(\llbracket \text{NP} \rrbracket) \mid \mathbf{v} \text{ outward to } \llbracket \text{NP} \rrbracket \}$

The notions of inward/outward vectors with respect to a reference object will not be defined here. The illustration in (30) will have to suffice. The prepositions *op* (on) and *bij* (near) can be defined in terms of the length  $|\mathbf{v}|$  of the vector  $\mathbf{v}$ :

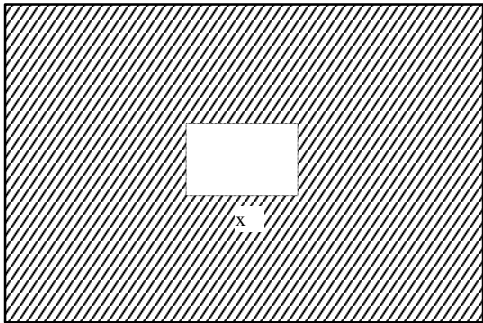
- (39) a.  $\llbracket \text{op NP} \rrbracket = \{ \mathbf{v} \in \text{space}(\llbracket \text{NP} \rrbracket) \mid |\mathbf{v}| \approx 0 \}$
- b.  $\llbracket \text{bij NP} \rrbracket = \{ \mathbf{v} \in \text{space}(\llbracket \text{NP} \rrbracket) \mid |\mathbf{v}| < r \} (r > 0)$

The diagrams (40) to (43) indicate what kind of regions are determined by these definitions. Notice however, that the diagrams only give a bounded two-dimensional cross-sections of regions that are three-dimensional and sometimes unbounded. The region is represented by the shaded area (unless the region is very small or thin, as in (42)) and the dotted line around an area indicates that the area is topologically open.<sup>12</sup>

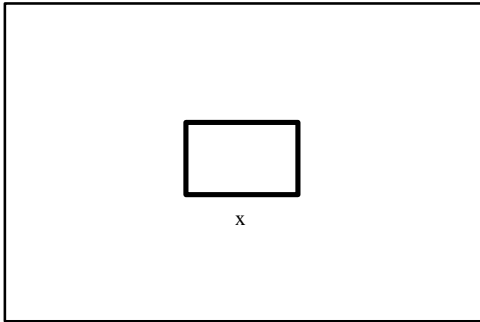
(40) *in / binnen x*



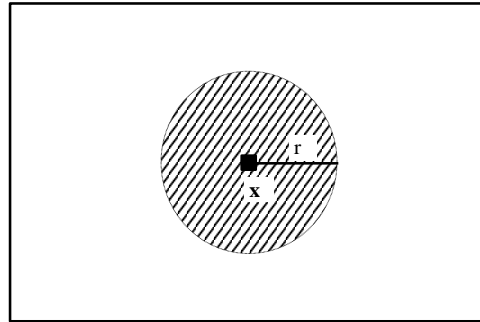
(41) *buiten x*



(42) *op* x



(43) *bij* x



Of course, these are only very rough and idealized representations of the meanings of these items. They capture the central intuitions that we seem to have about them, but it is obvious that many of the subtle and interesting complexities of use, discussed in Herskovits (1986), Vandeloise (1991), Cuyckens (1991), and others are ignored. How to capture the full range of meanings of prepositions like *in* and *op* clearly goes beyond the scope of this paper. Moreover, the prepositions *in* and *binnen* have received the same definition, in spite of obvious differences, like those in (44):

- (44) a.    *in* het water            vs.    \**binnen* het water  
          *in* the water            vs.    \**inside* the water
- b.    \**in* de grens                vs.    *binnen* de grens  
          \**in* the border        vs.    *inside* the border

What exactly distinguishes *binnen* from *in* must be left for further research. The same is true for the factors that may determine the ‘radius’ *r* in the definition of *bij* in (39b), like the size of the reference object, the structure of the environment, etc.

The regions denoted by the prepositions in (37) (except for *tussen*) are determined by an *axis*. There are three orthogonal axes.<sup>13</sup> The vertical up-down axis is determined by the line

of gravitation usually. The horizontal front-back axis can be *intrinsic* to the reference object in virtue of its shape, movement, or function) or assigned *deictically* (from the point of view of an observer). The *lateral* axis goes side-to-side, is perpendicular to the other two axes, and has a left-hand side and a right-hand side. Again I will abstract away from the many different factors that may determine these axes and simply assume that the model provides three axes, or rather half-axes, each defined as sets of vectors:<sup>14</sup>

- (45) VERT            the set of vectors pointing upward  
       FRONT         the set of vectors pointing forward  
       DEXT          the set of vectors pointing rightward

Of course these half-axes correspond to the prepositions *boven* (above), *voor* (in front of), and *rechts van* (to the right of). The respective antonyms of these prepositions, *onder* (below), *achter* (behind), and *links van* (to the left of) correspond to the *inverses* of the axes in (45). The inverse of an axis (and of a set of vectors in general) is simply the set of vectors pointing in the opposite direction:

- (46) the *inverse* of an axis  $A$  is  $-A = \{ \mathbf{v} \in \mathbf{S} \mid -\mathbf{v} \in A \}$

Another useful notion is that of the so-called *orthogonal complement*  $\perp A$ , which is the set of vectors orthogonal to the vectors in an axis or plane  $A$ :

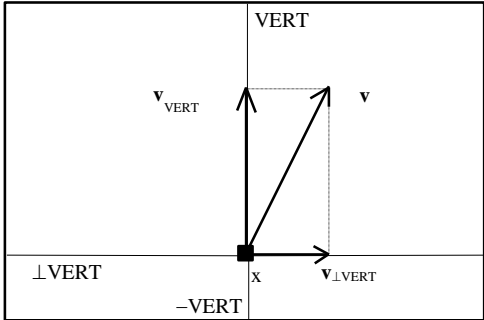
- (47) the *orthogonal complement* of  $A$  is  $\perp A = \{ \mathbf{v} \in \mathbf{S} \mid \forall \mathbf{w} \in A [ \mathbf{v} \perp \mathbf{w} ] \}$



For example, the orthogonal complement  $\perp\text{VERT}$  of the upward vertical axis  $\text{VERT}$  is the set of horizontal vectors and the lateral axis  $\text{LAT}$  corresponding to *naast* (beside) can be defined as the orthogonal complement of  $\text{VERT} \cup \text{FRONT}$ .

Given these axes, every vector can be decomposed into several *components* in the axes. This is illustrated in (48). The vector  $\mathbf{v}$  is decomposed into a vertical component  $\mathbf{v}_{\text{VERT}}$  (its *projection* on the vertical axis) and a horizontal component  $\mathbf{v}_{\perp\text{VERT}}$  (its projection on the orthogonal complement of the vertical axis).

(48) *Axes and projections*



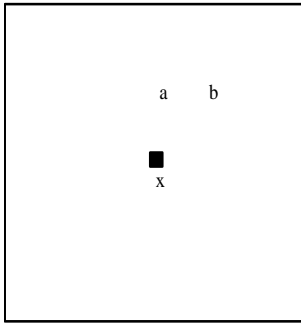
The notion of projection used here is defined in (49):

(49) The *projection*  $\mathbf{v}_A$  of vector  $\mathbf{v}$  on axis  $A$  is that vector  $\mathbf{u} \in A$  for which there is a  $\mathbf{w}$ , such that  $\mathbf{w} \perp \mathbf{u}$  and  $\mathbf{u} + \mathbf{w} = \mathbf{v}$

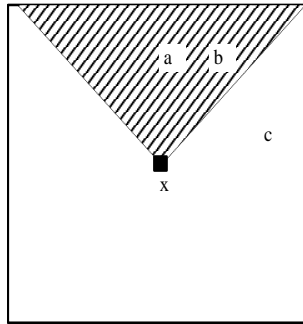
This provides the necessary formal apparatus to define the most important direction-based prepositions. Herskovits (1986) shows that the size of the region corresponding to a preposition like *boven* (above) can vary:

(50)

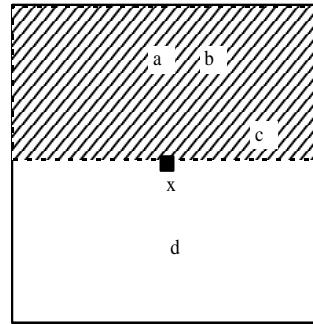
a. *boven x*



b. *boven x*



c. *boven x*



In (50a), we can say that a is above x, but b is not, but in (50b), both a and b are above x, but c is not. However, when comparing a, b, and c with d in (50c), we can say that a, b, and c are above x, but d is not. So the region corresponding to *above* in a particular situation depends on the contrast we want to make.

All of these regions can be expressed as sets of vectors emanating from the reference object x. For situation (50a), it is simply the axis VERT that is taken as the above-region, for (50b) it is the set of those vectors such that the projection on the vertical axis has a greater length than the projection on the orthogonal horizontal plane. The vector  $\mathbf{v}$  in (48) is an example of a vector that satisfies this condition.. For (50c), only those vectors count that have a non-zero component on the VERT-axis. What is common to these definitions is that they all impose conditions on the length of the vertical component:

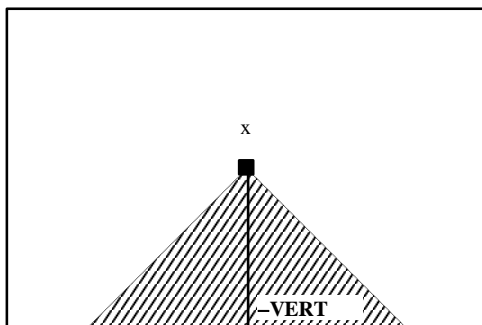
- (51) a.  $[[\text{boven NP}]] = \{ \mathbf{v} \in \text{space}([\text{NP}]) \mid |\mathbf{v}_{\text{VERT}}| = |\mathbf{v}| \}$   
b.  $[[\text{boven NP}]] = \{ \mathbf{v} \in \text{space}([\text{NP}]) \mid |\mathbf{v}_{\text{VERT}}| > |\mathbf{v}_{\perp \text{VERT}}| \}$   
c.  $[[\text{boven NP}]] = \{ \mathbf{v} \in \text{space}([\text{NP}]) \mid |\mathbf{v}_{\text{VERT}}| > 0 \}$

For the purposes of this paper it is most convenient to assume that prepositions like *boven* denote cone-shaped regions defined as in (51b). The definitions of the other axis-based prepositions are very similar:

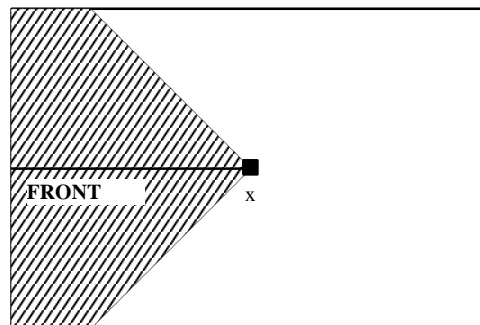
- (52) a.  $[[\text{onder NP}]] = \{ \mathbf{v} \in \text{space}([[\text{NP}]] ) \mid |\mathbf{v}_{\text{-VERT}}| > |\mathbf{v}_{\perp\text{-VERT}}| \}$   
 b.  $[[\text{voor NP}]] = \{ \mathbf{v} \in \text{space}([[\text{NP}]] ) \mid |\mathbf{v}_{\text{FRONT}}| > |\mathbf{v}_{\perp\text{FRONT}}| \}$   
 c.  $[[\text{achter NP}]] = \{ \mathbf{v} \in \text{space}([[\text{NP}]] ) \mid |\mathbf{v}_{\text{-FRONT}}| > |\mathbf{v}_{\perp\text{-FRONT}}| \}$   
 d.  $[[\text{rechts van NP}]] = \{ \mathbf{v} \in \text{space}([[\text{NP}]] ) \mid |\mathbf{v}_{\text{DEXT}}| > |\mathbf{v}_{\perp\text{DEXT}}| \}$   
 e.  $[[\text{links van NP}]] = \{ \mathbf{v} \in \text{space}([[\text{NP}]] ) \mid |\mathbf{v}_{\text{-DEXT}}| > |\mathbf{v}_{\perp\text{-DEXT}}| \}$   
 f.  $[[\text{naast NP}]] = \{ \mathbf{v} \in \text{space}([[\text{NP}]] ) \mid |\mathbf{v}_{\text{LAT}}| > |\mathbf{v}_{\perp\text{LAT}}| \}$

The definitions in (52a)-(52e) yield regions like those in (53)-(55) and the definition of *naast* in (52f) corresponds to the diabolo-shaped region in (56). (Remember that the diagrams only give finite cross-sections of these regions.)

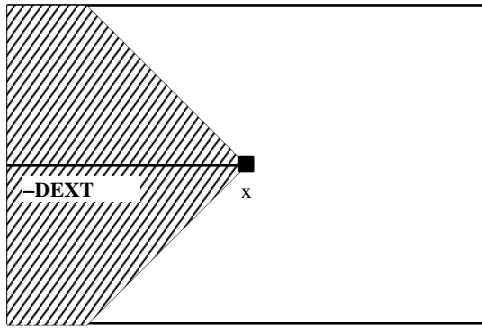
(53) *onder x (frontview)*



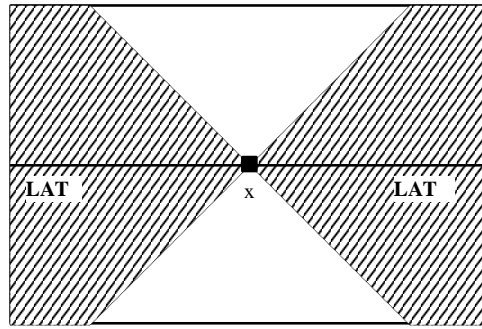
(54) *voor x (sideview)*



(55) *links van x (frontview)*

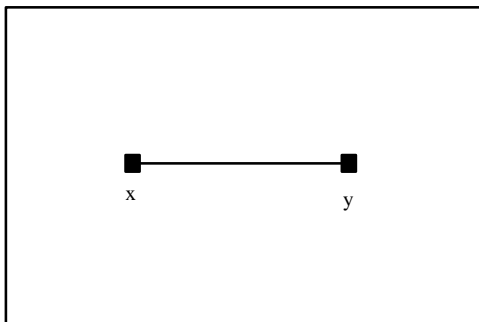


(56) *naast (frontview)*



The definition of *tussen* (between), finally, is of a slightly different nature. In order to keep things simple, the definition given here only covers those uses of *tussen* with two reference objects, where the region is a line between the two objects, as shown in (57):

(57) *tussen x en y*



The definition is given in (58):

(58)  $\llbracket \text{tussen NP}_1 \text{ en NP}_2 \rrbracket =$

$\{ \mathbf{v} \in \text{space}(\llbracket \text{NP}_1 \rrbracket) \mid \exists s [ s \geq 1 \wedge \text{loc}(\llbracket \text{NP}_2 \rrbracket, s\mathbf{v}) ] \} \cup$

$\{ \mathbf{v} \in \text{space}(\llbracket \text{NP}_2 \rrbracket) \mid \exists s [ s \geq 1 \wedge \text{loc}(\llbracket \text{NP}_1 \rrbracket, s\mathbf{v}) ] \}$

Essentially, a vector located at  $x$  is between  $x$  and another object  $y$  if it would end in  $y$  when lengthened. This is sufficient for our purpose, but see Habel (1989) for an extensive discussion of the semantics of German *zwischen* that covers many of the intricacies of this relation.

## 4.2 Definitions of modifiers

Section 2 indicated how the introduction of vectors makes it possible to interpret modifiers as functions that map the denotation of a PP into a subset:

$$(59) \quad \llbracket [\text{PP Mod PP}] \rrbracket_M = \{ \mathbf{v} \in \llbracket \text{PP} \rrbracket_M \mid \dots \mathbf{v} \dots \}$$

Each modifier imposes certain conditions on the length or direction of the vector  $\mathbf{v}$  in (59) that will be explained in somewhat more detail here.

The length of the vector can be specified in absolute terms by a measure phrase, for instance *twee meter* (two meters):

$$(60) \quad \llbracket [\text{twee meter PP}] \rrbracket = \{ \mathbf{v} \in \llbracket \text{PP} \rrbracket \mid |\mathbf{v}| = 2m \}$$

where  $m$  is a real number representing the unit of the meter. There is no need to treat the measure phrase as the specifier of an invisible adjective *far*, as proposed in Wunderlich & Kaufmann (1990:241) and Wunderlich & Herweg (1991:780), because the measure phrase can directly apply to the PP denotation. This is also empirically more adequate, because in Dutch some measure phrases are possible with PPs even when they cannot be combined with a dimensional adjective:

- (61) a. een eind buiten de stad  
 a long distance (lit. end) outside the city
- b. \* een eind ver/hoog/diep  
 a long distance far/high/deep

Adverbs like *vlak* (right) and *direct* (directly) express that the length is almost zero:

$$(62) \quad \llbracket \text{vlak PP} \rrbracket = \llbracket \text{direct PP} \rrbracket = \{ \mathbf{v} \in \llbracket \text{PP} \rrbracket \mid |\mathbf{v}| \approx 0 \}$$

The adjectives *ver* (far) and *dicht* (close) specify the length of the vector in relative terms, by comparing it with a contextually given norm  $r$ :

$$(63) \quad \llbracket \text{ver PP} \rrbracket = \{ \mathbf{v} \in \llbracket \text{PP} \rrbracket \mid |\mathbf{v}| > r \}$$

$$\llbracket \text{dicht PP} \rrbracket = \{ \mathbf{v} \in \llbracket \text{PP} \rrbracket \mid |\mathbf{v}| < r \}$$

The same is true for the the adjectives *hoog* (high), *laag* (low), and *diep* (deep) but these adjectives carry additional information about direction, specifying that the vectors are downward (in the case of *diep*) or upward (in the case of *hoog* and *laag*):<sup>15</sup>

$$(64) \quad \text{a.} \quad \llbracket \text{hoog PP} \rrbracket = \{ \mathbf{v} \in \llbracket \text{PP} \rrbracket \mid \mathbf{v} \in \text{VERT} \wedge |\mathbf{v}| > r \}$$

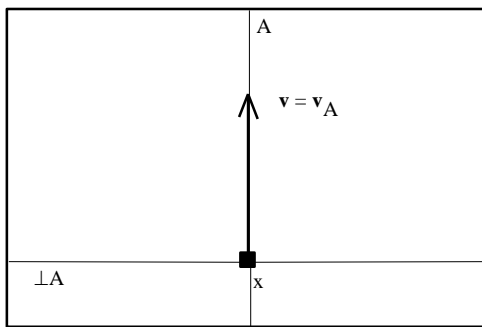
$$\text{b.} \quad \llbracket \text{laag PP} \rrbracket = \{ \mathbf{v} \in \llbracket \text{PP} \rrbracket \mid \mathbf{v} \in \text{VERT} \wedge |\mathbf{v}| < r \}$$

$$\text{c.} \quad \llbracket \text{diep PP} \rrbracket = \{ \mathbf{v} \in \llbracket \text{PP} \rrbracket \mid \mathbf{v} \in -\text{VERT} \wedge |\mathbf{v}| > r \}$$

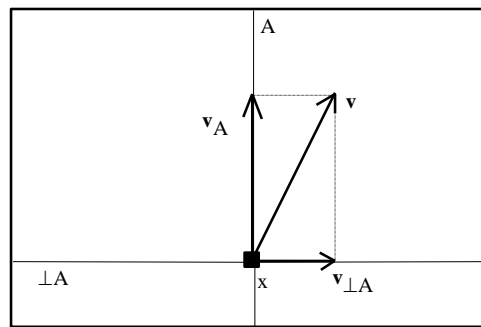
The modifiers *recht* and *pal* (straight) both express that a vector  $\mathbf{v}$  coincides with an axis that is taken as a reference axis (for example the vertical axis VERT), as illustrated in (65a). The modifier *schuin* (diagonally) indicates that the vector  $\mathbf{v}$  deviates from the reference axis and can be composed in two non-zero orthogonal components (65b).

(65)

a. *recht, pal*



b. *schuin*



The crucial difference is that in (65a) there is no orthogonal component, but in (65b) there is:

$$(66) \quad \llbracket \text{recht PP} \rrbracket = \llbracket \text{pal PP} \rrbracket = \{ \mathbf{v} \in \llbracket \text{PP} \rrbracket \mid |\mathbf{v}_{\perp A}| = 0 \}$$

$$\llbracket \text{schuin PP} \rrbracket = \{ \mathbf{v} \in \llbracket \text{PP} \rrbracket \mid |\mathbf{v}_{\perp A}| > 0 \}$$

Finally, the modifiers *rechts* (right) and *links* (left) are defined in terms of the DEXT and  $-\text{DEXT}$  axes:

$$(67) \quad \llbracket \text{rechts PP} \rrbracket = \{ \mathbf{v} \in \llbracket \text{PP} \rrbracket \mid |\mathbf{v}_{\text{DEXT}}| > 0 \}$$

$$\llbracket \text{links PP} \rrbracket = \{ \mathbf{v} \in \llbracket \text{PP} \rrbracket \mid |\mathbf{v}_{-\text{DEXT}}| > 0 \}$$

*Rechts* selects the vectors that have a component on the rightward axis and *links* selects the vectors that have a component on the leftward axis.

The vector interpretation of modifiers makes it possible to relate the modifier use of dimensional adjectives like *hoog* and *laag* and directional modifiers like *recht* and *schuin* to their more basic adjectival uses, like in the following examples (with literal English translations):

- (68)
- a. een hoge toren  
a high tower
  - b. een lage toren  
a low tower
  - c. een rechte toren  
a straight tower
  - d. een schuine toren  
a slanting tower

In these examples, the modifier picks out a subset of those objects that have a particular size or orientation with respect to the vertical axis. Suppose now that certain objects are associated with a vector that represents its main axis (like in the visual 3D models of Marr 1982). A tower, for example, will be associated with an upward vector that represents its main vertical axis. The definitions that we gave for the PP modifier use of *hoog*, *laag*, *recht* en *schuin* can then easily be adapted for the adjectival use. The basic condition that is imposed on the length or direction of the vector is the same in both uses. A tower is *hoog* if its vector  $\mathbf{v}$  satisfies the condition  $\mathbf{v} \in \text{VERT} \wedge |\mathbf{v}| > r$  and it is *recht* if the component in the horizontal plane is zero, i.e.  $|\mathbf{v}_{\perp \text{VERT}}| = 0$ . Although I am not able to provide a full treatment of such cases here and to



do justice to the richness of dimensional and directional adjectives, these examples suggest that a semantics based on vectors can be extended naturally from locative prepositions to other expressions with a spatial meaning component.

## **5 Closure and continuity properties of regions**

The regions denoted by locative PPs (whether modified or not) have certain formal properties that crucially rely on their vector structure. A region may be closed under operations of scalar multiplication and vector addition and a region may or may not have certain continuity properties.<sup>16</sup> I will explain such properties in this section and show their importance for the semantics of PPs. Some of the properties seem to express general constraints on prepositional denotations ('semantic universals'), others define natural classes of prepositions with a characteristic semantic behaviour. It is crucial that *all* locative prepositions are interpreted in terms of vectors and not just those that involve axes or directions. It is only because all locative prepositions are of the same vector type that we can sharply formulate such properties as introduced in this section and discover where the sets of vectors denoted by, for example, *inside*-PPs and *behind*-PPs differ and agree. Compare this to the theory of Generalized Quantifiers, where the uniform treatment of all noun phrases as sets of sets is the basis for a fruitful research program.

### **5.1 Closure properties**

As explained in section 3, a vector can be multiplied by a scalar in order to change its length. If the scalar is greater than 1, the vector will be *lengthened*, if it is between 0 and 1, the vector will be *shortened*. Lengthening and shortening can be seen as operations on vectors in a

region and regions may or may not be closed under these operations. Closure under lengthening can be defined as follows:

(69) *Closure under lengthening*

A region  $R$  is closed under lengthening iff

for every non-zero  $\mathbf{v} \in R$ ,  $s\mathbf{v} \in R$  for every  $s > 1$ .

Given an arbitrary vector  $\mathbf{v}$  in a region that is closed under lengthening, one can stretch this vector and the result will still be in the region. Intuitively, a region that is closed under lengthening is unbounded in the direction in which the vectors point. A region that is not closed under lengthening is bounded, because of the preposition used (e.g. *bij*), because the reference object is bounded (e.g. with *binnen*) or because of modifiers like *dicht* (close) and *hoogstens twee meter* (at most two meters):<sup>17</sup>

(70) <i>closed under lengthening</i>	<i>not closed under lengthening</i>
voor (in front of)	bij (near)
achter (behind)	in, binnen (in, inside)
boven (above)	op (on, at)
onder (under)	tussen (between)
naast (beside)	
buiten (outside)	
ver achter (far behind)	dicht achter (close behind)
minstens 2 m achter	hoogstens 2 m achter
(at least two meters behind)	(at most two meters behind)

This distinction is significant, because the PPs that are closed under lengthening are the ones that can be modified by measure phrases like *two meters*, provided that they are not already modified. This is illustrated by the following examples:<sup>18</sup>

- (71) a. twee meter voor / achter / boven / onder / naast / buiten NP  
 two meters in front of / behind / above / under / beside / outside NP
- b. \* twee meter tussen / bij / in / op / binnen NP  
 two meters between / near / in / on / inside NP

This shows that measure phrase modifiers are not just functions from regions to subregions, but they apply only to regions that are closed under lengthening. The intuitive explanation is that measure phrases specify a value or range of values on an open-ended scale and the regions that are closed under lengthening are the regions that provide such a scale. Notice that the examples in (71) show that it will not do to say that measure phrases measure a distance on the spatial *axis* (like VERT, FRONT, DEXT) associated with a projective preposition, which would obviate the need for vectors.<sup>19</sup> PPs with *buiten* (outside) can also be modified by means of a measure phrase, even though there is no axis.

Closure under shortening is like closure under lengthening:

(72) *Closure under shortening*

A region  $R$  is closed under shortening iff

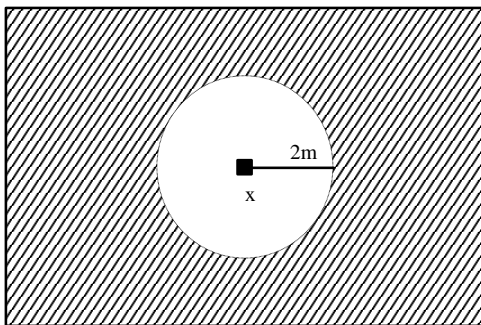
for every  $\mathbf{v} \in R$ ,  $s\mathbf{v} \in R$  for every  $0 < s < 1$ .

This property says that one can take an arbitrary vector from the region, make it shorter, and the result will again be in the region.

(73)	<i>closed under shortening</i>	<i>not closed under shortening</i>
	all simple prepositions	
	recht boven (straight above)	
	laag boven (low above)	hoog boven (high above)
	minder dan 2 m buiten	meer dan 2 m buiten
	(less than 2 m outside)	(more than 2 m outside)

Intuitively, the regions that are closed under shortening make contact with the (surface of the) reference object. The diagram in (74) shows the region of *meer dan twee meter buiten x* (more than two meters outside *x*). The region does not make contact with the reference object, because there is a gap of two meters created by the modifier *meer dan twee meter* (more than two meters).

(74) *meer dan twee meter buiten x*



Notice that in (73) all *simple* PPs (i.e. PPs without modifiers) are closed under shortening. This might be taken as a coincidence, but it would be more interesting to interpret it as a constraint on prepositional semantics:

(75) *Universal 1* All simple PPs are closed under shortening.

This universal implies that simple PPs denote regions that overlap with every environment of the reference object. One clear and easily falsifiable prediction drawn from this universal is that there are no *distal* prepositions. This seems to be true for Dutch and English (and other languages I know of): the proximate PP *bij NP* (near NP) does not have a distal counterpart *ver NP* (far NP). Of course, this gap is filled by the expression *ver van NP* (far from NP), but this is not a simple PP, since it involves a combination of the *directional* preposition *van* (from) with the distal adjective *far* (ver).<sup>20</sup> The universal would only be falsified by a distal expression which is a genuine simple PP.

Some PPs have the property of closure under vector addition:

(76) *Closure under vector addition*

A region  $R$  is closed under vector addition iff

for every  $\mathbf{v}, \mathbf{w} \in R$ ,  $\mathbf{v} + \mathbf{w} \in R$ .

As shown in (77), this property, which is characteristic for direction-based prepositions like *boven* (above) and *achter* (behind), can be lost with certain modifiers:

(77) *Closed under addition*

*Not closed under addition*

boven (above)

recht boven (straight above)

hoog boven (high above)

minstens 2m boven

(at least 2m above)

schuin boven (diagonally above)

vlak boven (right above)

hoogstens 2m boven

(at most 2m above)

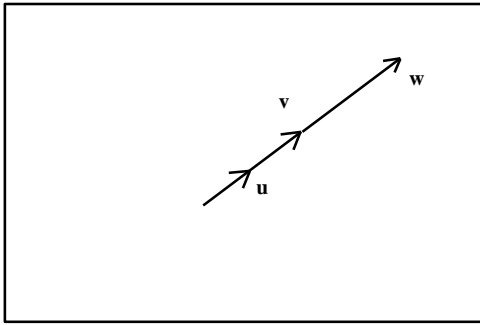
The property of closure under addition corresponds with the *transitivity* of the underlying prepositional relation. This is why (78c) follows from the premises (78a) and (78b), while (79c) can not be concluded from (79a) and (79b) (talking about paintings on a wall):

- (78) a. Vermeer hangt (recht/hoog) boven Rembrandt  
Vermeer is hanging straight/high above Rembrandt
- b. Rembrandt hangt (recht/hoog) boven Van Steen  
Rembrandt is hanging straight/high above Van Steen
- c. Vermeer hangt (recht/hoog) boven Van Steen  
Vermeer is hanging straight/high above Van Steen
- (79) a. Vermeer hangt schuin/vlak boven Rembrandt  
Vermeer is hanging diagonally/right above Rembrandt
- b. Rembrandt hangt schuin/vlak boven Van Steen  
Rembrandt is hanging diagonally/right above Van Steen
- c. Vermeer hangt schuin/vlak boven Van Steen  
Vermeer is hanging diagonally/right above Van Steen

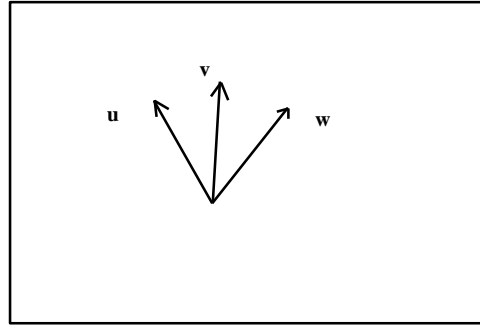
## 5.2 Continuity properties

The region denoted by a locative PP can be continuous or discontinuous. Continuity can be defined in many ways, but the definitions used here are based on the two ways in which a vector  $\mathbf{v}$  can be ‘between’ two other vectors  $\mathbf{u}$  and  $\mathbf{w}$ :<sup>21</sup>

(80)  $\mathbf{v}$  linearly between  $\mathbf{u}$  and  $\mathbf{w}$



(81)  $\mathbf{v}$  radially between  $\mathbf{u}$  and  $\mathbf{w}$



A vector  $\mathbf{v}$  is *linearly between*  $\mathbf{u}$  and  $\mathbf{w}$  if  $\mathbf{v}$  is a lengthening of  $\mathbf{u}$  and  $\mathbf{w}$  is a lengthening of  $\mathbf{v}$ .

A vector  $\mathbf{v}$  is *radially between* two vectors  $\mathbf{u}$  and  $\mathbf{w}$  that form an acute angle if the shortest rotation of  $\mathbf{u}$  into  $\mathbf{w}$  passes over  $\mathbf{v}$ . Both of these notions of ‘betweenness’ correspond to a form of continuity:

(82) a. *Linear continuity*

A region  $R$  is linearly continuous iff

for all  $\mathbf{u}, \mathbf{w} \in R$ , if  $\mathbf{v}$  is linearly between  $\mathbf{u}$  and  $\mathbf{w}$ , then  $\mathbf{v} \in R$

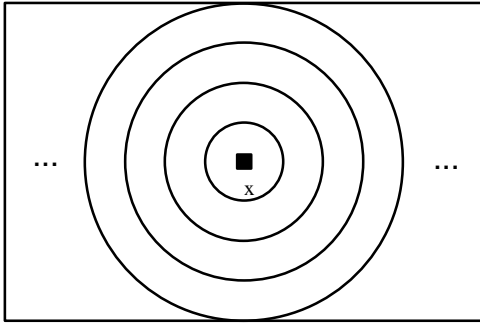
b. *Radial continuity*

A region  $R$  is radially continuous iff

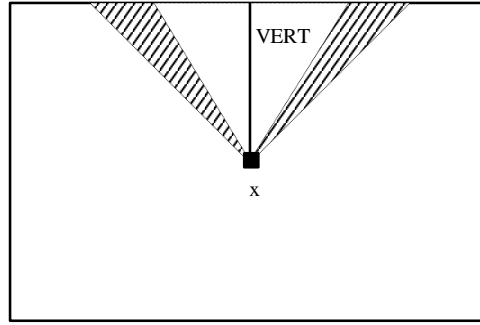
for all  $\mathbf{u}, \mathbf{w} \in R$ , if  $\mathbf{v}$  is radially between  $\mathbf{u}$  and  $\mathbf{w}$ , then  $\mathbf{v} \in R$

The best way to grasp these continuities is by looking at two cases that each lack one of these properties. The region in (83) denoted by *een even aantal meters buiten x* (an even number of meters outside  $x$ ) consists of an infinite set of concentric shells around the reference object  $x$ . This region is *radially* continuous but *linearly discontinuous*. The *PP schuin boven x* (diagonally above  $x$ ) in (84) denotes a region that is *linearly* continuous, but *radially discontinuous*.

(83) *een even aantal meters buiten x*



(84) *schuin boven x*



Two continuity universals can now be formulated, a stronger one for *simple* PPs and a weaker one for all PPs, whether they are modified or not:<sup>22</sup>

(85) *Universal 2* All simple PPs are linearly *and* radially continuous.

(86) *Universal 3* All PPs are linearly *or* radially continuous.

## Conclusion

The primary goal of this paper was to give a compositional and natural account of the interpretation of modified PPs. I have shown that both prepositions and modifiers of PPs can be interpreted in terms of vectors. The insights from the literature about prepositions and other elements (e.g. dimensional adjectives) can be integrated in a general, formal framework. The vector-algebraic background of this framework makes it possible to study the meanings of PPs, both simple and modified, in a way that is reminiscent of the Generalized Quantifier Theory of NPs: in addition to precise definitions of individual PPs, algebraic properties can be formulated that either single out empirically relevant subclasses of locative PPs or that provide (universal?) constraints on their denotation.



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## Notes

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Martijn Spaan, Henk Verkuyl, and Emile van der Zee for their comments on earlier written versions (with the usual disclaimers), the anonymous reviewers of this journal, and especially Yoad Winter for extensive discussion and fruitful cooperation.

2 See for example Bennett (1975), Cresswell (1978), Jackendoff (1983), and Lakoff (1987).

3 See Jackendoff (1983), Creary et al. (1989), Wunderlich (1991).

4 Alternatively, regions could be seen as primitives ('individuals') in a mereology of space, as in Bierwisch (1988), Aurnague (1995), and Nam (1995), for instance. A modifier then maps a region to a proper subregion, but since both argument and result of the modification function are unanalyzed, it is hard to specify what the modifier does, unless we have an idea of the mereological and geometric structuring of the spatial domain.

5 That this pronoun is not a clitic incorporated into the preposition is shown by its ability to switch places with the modifier: *er twee centimeter achter*, *daar hoog boven*, and by the possibility to move it out of the PP to several positions in the sentence, including the topic position (Van Riemsdijk 1978).

6 This picture of a PP region and the ones to follow are given for clarification only; they have no theoretical status of their own, unlike the diagrams used in the Cognitive Grammar literature (Langacker 1986, Hawkins 1984).

7 This formulation was suggested by an anonymous reviewer.

8 A full discussion of the two-level approach to prepositions and PP modifiers would go beyond the scope of this paper.

9 Vectors are usually defined as pairs of points or tuples of real numbers in the mathematics literature, but in this paper I wish to remain neutral with respect to their status and treat them as if they were spatial primitives.

10 For simplicity, reference objects are assumed to be *convex*, i.e. they contain no discontinuities, indentations, or protrusions, but are basically like familiar geometric objects (spheres, cylinders, rectangles, etcetera). How to deal with non-convex objects or with objects that seem to have no boundary (like the universe) or vague boundaries (like mass objects) is a topic for future research.

11 See also Creary et al. (1989). Conjunction of PPs with different reference objects cannot be interpreted directly as intersection, because such PPs usually denote disjoint sets of vectors. This is because each PP is part of its own vector space that is not directly related to the spaces of other PPs.

12 From the topological point of view all of these regions are open sets. Two objects touch if their surface regions overlap. See also Hayes (1985).

13 See Herskovits (1986) for extensive discussion of axes and their relation to prepositions like those in (37) and also Lang (1990). The treatment given here can only be very sketchy.

14 In a more realistic account, these axes would have to be relativized to objects. For example, if  $x$  is a television, then  $FRONT(x)$  is the set of vectors reaching out from the screen, roughly. But if  $x$  is a tree, then we would need a second argument:  $FRONT(x,y)$  would then be the set of vectors pointing from the tree  $x$  towards an observer  $y$ . In this paper, the model is set up in such a way that all objects are oriented in the same way.

15 I am not yet sure whether it would perhaps be more adequate to say that these modifiers do not measure the length of the vector itself, but its projection on the vertical axis, i.e.  $|v_{VERT}| > r$ , etc. Moreover, definition (64c) only gives the ‘downward’ meaning of *diep*. However, we have already seen that *diep* also has an ‘inward’ meaning, as in *diep in de boom* (deep in the tree) that I will not account for here.

16 Another property that I will not discuss in this paper is *closure under rotation*: we can change the direction of a vector while keeping its length fixed and check whether the result is still in the region. This property only works when the reference object is idealized as a point. For instance, PPs with *bij* (near) are closed under rotation, PPs with *boven* (above) are not closed under rotation. This property might be relevant for the proper use of a modifier like *recht* (straight): *\*recht bij de deur* (straight near the door) vs. *recht boven de deur* (straight above the door). It would be interesting (but far beyond the scope of this paper) to relate the vector-based closure properties to geometric invariances: which properties expressed by prepositions are invariant under such operations as translation and rotation and under projective and topological transformations? See Crangle and Suppes (1989) for an interesting discussion of prepositions from this geometric perspective.

17 The definitions in this section apply to regions, but a PP can be said to be closed under lengthening when the region it denotes is closed under lengthening in every model. Similarly for the other properties. I will talk about PPs or prepositions having a particular property when it is strictly speaking a property of the corresponding region.

18 The pattern in Dutch is clearer than in English, where *beside* and *next to* have a meaning component of contact or proximity, absent in Dutch, which makes that they are not closed under lengthening. *Binnen* (inside) and *in* (in) may become closed under lengthening when the reference object itself is open-ended (e.g. *twee centimeter in de muur*, ‘two centimeters into the wall’).

19 That axes would be sufficient to explain the use of measure phrases was suggested by an anonymous reviewer, who referred to Wunderlich & Herweg (1991:780).

20 That *ver* (far) is really an adjective in this construction and not a preposition is shown by its modification possibilities: *erg ver van het huis* (very far from the house), *te ver van het huis* (too far from the house).

21 There is an interesting connection here with recent work of Peter Gärdenfors, who claims that natural properties (expressed by color terms and common nouns, for example) can be represented as *convex regions* in conceptual spaces, i.e. if points  $v_1$  and  $v_2$  are in the region, then every point *between*  $v_1$  and  $v_2$  is also in the region (Gärdenfors 1994). The continuity universals of this section, defined in terms of between-relations over vectors, impose a similar condition on the regions denoted by PPs.

22 Yoad Winter pointed out to me that this universal does not seem to be valid for those PPs that are modified by a *conjunction* of two modifiers: e.g. *schuin en een even aantal meters boven x* (diagonally and an even number of meters above x). This PP denotes a region which is neither linearly nor radially continuous, because it is the intersection of the regions in (83) and (84). Given this, Universal 3 should be reinterpreted as a universal for PPs that do not contain conjunctions and we would need another, more general universal that applies to all locative PPs, simple, modified, and conjoined.