

Chapter 7

Quantifiers and Scope

The purpose of this chapter is to provide a grammatical account of the contribution of quantified noun phrases to the meanings of phrases in which they occur. Logically, generalized quantifiers can be characterized as functions from properties into propositions, with determiners acting as functions from properties into generalized quantifiers. It remains to explain how quantifiers remain *in situ*, yet take semantic scope around an arbitrary amount of surrounding material. As a minimal illustration, consider the following sentences with quantified subjects and objects.

- (1) a. Every kid played with some toy.
b. Some kid broke every toy.

These sentences provide the simplest possible example of *scope ambiguity*. The first sentence has two readings distinguished by the *scope* of the quantifiers $\text{every}^2(\text{kid})$ and $\text{some}^2(\text{toy})$ (recall that the superscript 2 indicates a constant of generalized determiner type).

- (2) a. $\text{every}^2(\text{kid})(\lambda x.\text{some}^2(\text{toy})(\lambda y.\text{play}(y)(x)))$
b. $\text{some}^2(\text{toy})(\lambda y.\text{every}^2(\text{kid})(\lambda x.\text{play}(y)(x)))$

Under the first reading, (2)a, every kid may have played with a different toy. In this reading, *every kid* is said to take *wide scope* and *some toy narrow scope*. Under the second reading, (2)b, there must be some toy such that every kid played with that toy. Thus, the second reading, (2)b, where the object quantifier takes wide scope over the subject quantifier, entails the first reading. From this fact alone, it might be tempting to claim that (2)a is the only relevant reading, as its truth is entailed by the truth of the other proposed reading. But now notice that although there is also an entailment between the two readings of (1)b, it is the subject wide scope reading which entails the object wide scope reading. Similarly,

by negating the predicate, any entailment relations between quantifier scopings will be reversed. Furthermore, with other quantifiers, such as *many* and *most*, there is no entailment relationship between the two relative scopings. It is simply not sufficient to provide a quantifier narrow scope at the point at which it can be thought to act as an argument.

In this chapter, we extend the type-logical approach of the previous chapters to provide an adequate grammatical characterization of scoping phenomena. Before doing so, we first survey the two most historically significant approaches to quantifier scoping, Montague's quantifying-in scheme and Cooper's storage mechanism. We also briefly survey H. Hendriks' (1987) highly influential type-shifting approach to quantification. The first truly logical approach to quantification was provided by van Benthem (1986a), who introduced a variant of the Lambek calculus that allowed arguments (including quantifiers) to permute, thus allowing all possible quantifier scopings. Unfortunately, this system overgenerated in that it allowed the arguments to functions to be confused, deriving readings for sentences like *every kid owned a dog* in which there is a single dog that owns every kid!

Following the work of van Benthem and Hendriks, Moortgat (1990a) introduced an elegant type-logical solution to the puzzle of scope by introducing a scoping constructor to account for the non-local semantic behavior of quantifiers in a compositional fashion. Using Moortgat's scoping constructor, the permutability of quantifiers in terms of scope could be captured without losing sensitivity to which arguments go with which functions. The scoping constructor is intimately related to the extraction constructor \uparrow , also introduced by Moortgat, in that both enable binding at a distance. The distinguishing factor grammatically is that the binding quantifier itself remains *in situ*, marking the position of binding. In order to bind the position occupied by the quantifier, the scoping construct builds a derivation with an extracted element in much the same way as the extraction constructor \uparrow . Pereira (1990) independently introduced a similar deductive approach to quantifier scoping. Although Pereira was not working in the context of type-logical grammar, his approach was isomorphic to Moortgat's in the scope derivations it predicted for quantifiers scoping at the sentential level. But because Pereira did not introduce a general connective for quantifiers, nor did he integrate his approach into a general type logic for concatenation such as Lambek's categorial grammar, the resulting approach is somewhat less general. More recently, Dalrymple et al. (1995) have

incorporated an approach similar to van Benthem's and Pereira's into LFG. They synchronize functions and arguments by projecting LFG's syntactic representations into a set of linear-logic formulas, over which derivations in the style of van Benthem can be carried out without any danger of losing control of which arguments go with which functions. The multi-stratal LFG and linear logic approach is almost identical to the method we adopt here in terms of the derivations it allows, although there are subtle differences arising due to the generality of linear logic relative to our type logics.

In this chapter, we focus on singular quantifiers and determiners. Next, in Chapter 8, we provide an account of plurality in which scope plays a crucial role. As we will see in Chapter 9, the type-logical approach to scoping admits natural categorizations for other *in situ* binders, such as reflexive and reciprocal pronouns, as well as relative pronouns that participate in pied-piping.

7.1 Quantifying In

The most well known and widely studied approach to quantifier scoping phenomena is that of Montague, as embodied in his PTQ grammar (Montague 1973). In the PTQ fragment, Montague admits an infinite number of pronoun lexical entries. Below is a slight simplification of his scheme (he actually treated all noun phrases as denoting generalized quantifiers and type raised pronoun entries accordingly).

$$(3) \text{he}_n \Rightarrow x_n: np$$

To account for quantifier scoping, Montague adopted a general scheme of *quantifying in*, the simplest case of which can be translated into our notation as follows (assuming an atomic category gq for generalized quantifiers with our standard extensional semantic typing).

$$(4) \text{ if } e \cdot \text{he}_n \cdot e'' \Rightarrow \phi: s \text{ and } e' \Rightarrow \alpha: gq, \\ \text{ then } e \cdot e' \cdot e'' \Rightarrow \alpha(\lambda x_n. \phi): s \qquad \text{(Quantifying In)}$$

This rule can be illustrated by starting with the derivations in (5).

$$(5) \text{ a. } \text{he}_5 \text{ ran} \Rightarrow \text{run}(x_5): s \\ \text{ b. } \text{everyone} \Rightarrow \text{every}: gq$$

We can now quantify (5)b into (5)a to produce the following result.

(6) *everyone run* \Rightarrow $\text{every}(\lambda x_5.\text{run}(x_5)): s$

Here we matched the quantifying-in scheme with $e = \varepsilon$, $e' = \textit{everyone}$, and $e'' = \textit{ran}$.

By iteratively quantifying in, we can derive relative scope ambiguities. For instance, assume we have the following derived results.

(7) a. *he₄ likes he₇* \Rightarrow $\text{like}(x_7)(x_4)$

b. *someone* \Rightarrow $\text{some}: gq$

Then by quantifying into (7)a, we have the following derivations.

(8) a. *everyone likes he₇* \Rightarrow $\text{every}(\lambda x_4.\text{like}(x_7)(x_4)): s$,

b. *everyone likes someone* \Rightarrow
 $\text{some}(\lambda x_7.\text{every}(\lambda x_4.\text{like}(x_7)(x_4))): s$

(9) a. *he₄ likes someone* \Rightarrow $\text{some}(\lambda x_7.\text{like}(x_7)(x_4)): s$

b. *everyone likes someone* \Rightarrow
 $\text{every}(\lambda x_4.\text{some}(\lambda x_7.\text{like}(x_7)(x_4))): s$

In the first, we have reduced the subject and then the object, giving the subject widest scope. In the second derivation, the order is reversed.

In Chapter 9, we consider the full generality of Montague's quantifying-in scheme, which he also used to account for pronominal binding. There will also be ample opportunity to explore further consequences of the quantifying-in scheme, some of which are desirable and some of which are not, by way of comparison to the type-logical approach.

7.2 Cooper Storage

Before moving on to present the type-logical approach to scoping phenomena, we briefly describe the storage-based approach to scoping introduced and elaborated by Cooper (1975, 1979, 1983). One of Cooper's primary goals was to eliminate the operation of quantifying in from the syntactic domain, thus isolating the effects of quantification in the semantics.

Cooper generalized the notion of meaning along two dimensions. The first generalization enables quantifiers to apply nonlocally without modifying the structure of the syntactic derivations. That is, quantifier binding is transparent to the syntactic schemes that attach noun

<i>Content</i>	<i>Store</i>
like (y)(x)	x / every , y / some
every (λx . like (y)(x))	y / some
some (λy . like (y)(x))	x / every
some (λy . every (λx . like (y)(x)))	ε
every (λx . some (λy . like (y)(x)))	ε

Figure 7.1Cooper's Stored Quantifiers for *everyone likes someone*

phrases as arguments. To achieve this effect, Cooper assumed that a meaning consisted not only of a semantic term of the appropriate type, but also a record of quantifiers inherited through the derivation from subexpressions. More specifically, a meaning is taken to be a pair $\alpha; Q$ in which α is a λ -term of the appropriate semantic type (perhaps with free variables), and where Q is a (possibly empty) sequence of variable/quantifier pairs, known as the *store*. The variables paired with the quantifiers indicate which variables they bind.

For example, *everyone likes someone* is analyzed syntactically as a sentence, with the set of meanings in Figure 7.1. The first meaning has both quantifiers in storage, the next two have one of the quantifiers in storage, and the last two have no quantifiers in storage. Thus only the last two count as readings of the sentence as a whole; the first three are incomplete with respect to the resolution of quantifier scope.

Cooper's second generalization was to associate each syntactic analysis not with a single meaning, but with a set of meanings, each of which consisted of a term and a store, as above. This move is significant in that it allows a single phrase structure scheme to behave nondeterministically, producing a set of results representing the possible ambiguities induced by quantification. With these two generalizations, Cooper was able to generalize the semantics attached to phrase structure schemes in order to generate all of the readings in Figure 7.1 with a single syntactic analysis.

Cooper's grammar can be generated by starting with an ordinary phrase structure grammar that associates single meanings to categories. Such a grammar can be extended to handle first stores and then sets of meanings. For illustration, we provide a pair of rules that construct a sentence out of a transitive verb and two arguments. These two rule schemes were first introduced by Montague (1973), albeit in a more general, intensional setting.

- (10) a. $x: gq \ y: vp \Rightarrow x(y): s$
 b. $x: tv \ y: gq \Rightarrow x(y): vp$

These schemes illustrate the way in which both Cooper and Montague type lift noun phrase arguments to be quantifiers (and hence why Montague's entries for pronouns he_n had to be assigned the type-raised quantifier term $\lambda P.P(x_n)$). The types can be induced from the terms and the type of generalized quantifiers; transitive verbs apply to their quantified objects to produce verb phrases, and subjects apply to verb phrases to produce sentences. Thus we have vp assigned type $\mathbf{Ind} \rightarrow \mathbf{Bool}$, gq assigned to $(\mathbf{Ind} \rightarrow \mathbf{Bool}) \rightarrow \mathbf{Bool}$, and tv assigned to $((\mathbf{Ind} \rightarrow \mathbf{Bool}) \rightarrow \mathbf{Bool}) \rightarrow \mathbf{Ind} \rightarrow \mathbf{Bool}$.

In our presentation of Cooper's grammar rule schemes, the daughter categories are assigned a variable semantics, and the mothers a compound term constructed out of the daughter's semantics. Enforcing this assumption in general leads to grammars which are wholly syntax-driven. That is, a rule can never fail to fire on the basis of its daughter's failing to match the input semantically. Furthermore, this form of rule allows the mother's semantics to be computed by variable substitution. For instance, with daughters $\alpha: gq$ and $\beta: vp$, the output produced by applying the scheme (10)a is $x(y)[x \mapsto \alpha][y \mapsto \beta] \stackrel{\text{def}}{=} \alpha(\beta)$, as is to be expected. Such a strategy follows Bach's (1989) notion of *rule-to-rule compositionality*, in which each grammar rule provides a combinator which computes the semantic term assigned to the mother as a function of the semantic terms assigned to the daughters. The combinator in the case of (10)a is $\lambda x.\lambda y.x(y)$; applying this to the terms assigned to the ordered sequence of daughters determines the semantics assigned to the mother.

As it stands, the phrase structure schemes in (10) can only be used to scope quantifiers as they are consumed as arguments, as shown in Figure 7.2. For extensional verbs such as *likes*, we have the meaning postulate in (11)a, which was first introduced by Montague (1973).

- (11) a. $\text{like} \stackrel{\text{def}}{=} \lambda Q.\lambda x.Q(\lambda y.\text{like}_p(y)(x))$
 b. $\text{some}(\text{like}(\text{every})) \equiv \text{some}(\lambda x.\text{every}(\lambda y.\text{like}_p(y)(x)))$

Applying substitution and the postulate in (11)a leads to the reduction in (11)b.

The first generalization of Cooper's extends rules such as those in (10) to allow for meanings that consist of term/store pairs. This is achieved

$$\begin{array}{c}
\frac{\textit{someone}}{\textit{some: } gq} \quad \frac{\textit{likes}}{\textit{like: } tv} \quad \frac{\textit{everyone}}{\textit{every: } gq} \\
\hline
\textit{like}(\textit{every}): vp \\
\hline
\textit{some}(\textit{like}(\textit{every})): s
\end{array}$$

Figure 7.2
In Situ Quantifier Binding

as follows.

- (12) if $x_1:C_1 \cdots x_n:C_n \Rightarrow \alpha:C_0$,
then $x_1; Q_1:C_1 \cdots x_n; Q_n:C_n \Rightarrow \alpha; Q_1 \cdots Q_n:C_0$ (Percolation)

Here we have used Q_i as a variable ranging over sequences of variable/quantifier term pairs. Note that the store associated with the mother category is simply the concatenation of the stores associated with the daughter categories. Further note that in a phrase structure approach, the order of concatenation results in the order of quantifiers in the store reflecting their linear order in the expression being analyzed. Cooper used this latter fact to prevent certain cases of backward-looking anaphora.

In order to extend the phrase structure rules to sets, two important closure conditions are enforced. These two conditions allow quantifiers to be stored and retrieved. Percolating a quantifier in storage through the derivation to the point at which it is retrieved is already handled by (12). The first closure condition affects sets of meanings containing quantifiers. The second affects sets of meanings containing propositions.

- (13) A set S of meanings is *closed* if and only if the following two conditions are satisfied:

- a. if $\alpha; Q \in S$, α of type $(\mathbf{Ind} \rightarrow \mathbf{Bool}) \rightarrow \mathbf{Bool}$,
then $\lambda P.P(x); x/\alpha \cdot Q \in S$. (Storage)
- b. if $\alpha; Q_1 \cdot x/\beta \cdot Q_2 \in S$, α of type \mathbf{Bool} ,
then $\beta(\lambda x.\alpha); Q_1 \cdot Q_2 \in S$. (Scoping)

For example, by the storage condition, if S is closed and $\textit{every}; \varepsilon \in S$, then $\lambda P.P(x); x/\textit{every} \in S$, too. Furthermore, according to the scoping scheme, if $\textit{run}(x); x/\textit{every} \in S$ and S is closed, then $\textit{every}(\lambda x.\textit{run}(x)); \varepsilon \in S$. As usual, we can close an arbitrary set S

$$\begin{array}{c}
\begin{array}{ccc}
\textit{someone} & \textit{likes} & \textit{everyone} \\
\hline
\lambda V.V(y); y/\textit{some}: gq & \textit{like}: tv & \lambda U.U(x); x/\textit{every}: gq \\
\hline
& \textit{like}(\lambda U.U(x)); x/\textit{every}: vp & \\
\hline
(\lambda V.V(y))(\textit{like}(\lambda U.U(x))); y/\textit{some} \cdot x/\textit{every}: s
\end{array}
\end{array}$$

Figure 7.3
Cooper Storage Derivation

of meanings by defining $Close(S) = T$ if and only if T is the least set such that $S \subseteq T$ and T is closed. Thus we can think of $Close(T)$ as freely applying quantifier scoping and storage.

Finally, to extend the phrase structure rules which percolate stores to the case of sets of meanings, we simply apply the rules pointwise to the input sets, and close the result under the storage and scoping schemes.

- (14) if $M_1:C_1 \cdots M_n:C_n \Rightarrow \Phi:C_0$,
then $S_1:C_1 \cdots S_n:C_n \Rightarrow Close(\{\Phi[M_i \mapsto \alpha_i] \mid \alpha_i \in S_i\}):C_0$

Here we have used the variables M_i to range over term/store pairs, and the variables S_i to range over sets of such term/store pairs. This scheme basically says that a term/store pair appears in the mother category's meaning set if it can be derived by applying a rule to a sequence of meanings selected from the daughter's meaning sets. Furthermore, closure applies to generate new meanings by storage and scoping from this basic set of meanings; these derived meanings must also appear in the set of meanings assigned to the mother.

In Figure 7.3, we provide a sample derivation where we have only included one member of the set of meanings at each node, and have not performed any reductions on the terms. The storage closure condition on quantifiers provided the derivations of the quantifiers. Given the meaning postulate for **like** in (11), the semantic term assigned to the sentential result in Figure 7.3 reduces to $\textit{like}_p(x)(y)$ by four applications of β -reduction. Application of the scoping closure condition on propositions produces the remaining readings in Figure 7.1.

Cooper employed several filters on his closure conditions to prevent free variables and vacuous binding. We will explore some of these restrictions below when comparing Cooper storage to Moortgat's type-logical account of quantification. In that context, we will also discuss a more representative range of quantificational facts than can be captured by means of storage.

7.3 Scoping Constructor

Many linguists seem to feel that if a semantic ambiguity has no distributional analogue, then its behavior should be characterized purely semantically. Because quantifiers share pretty much the same distribution as other noun phrases, issues of definiteness aside, Montague took the first opportunity to banish the syntactic distinction between quantifiers and other noun phrases. Because there seemed to be no syntactic distinction induced by the point at which a quantifier applied semantically, Cooper removed the syntactic structure associated with quantifying in, as well. Of course, we eschew this line of reasoning; in the type-logical approach, syntax is merely the vehicle by which expressions are associated with meanings. Thus it is not at all unnatural to find “syntactic” operations with “semantic” effects; they are merely two sides of the same coin.

As it turns out, the sequent presentation of the scoping constructor’s rule of use bears a striking resemblance to Montague’s quantifying-in scheme. Furthermore, the natural deduction presentation of the logic is remarkably similar to Cooper storage. These similarities run so deeply that the type-logical approach to scoping can be viewed as a rational reconstruction of the rather ad hoc approaches of Montague and Cooper (though, ironically, Moortgat’s scoping constructor was intended as a logical reconstruction of H. Hendriks’ (1987, 1993) type-shifting approach). In addition, the type-logical perspective on quantification sheds an enormous amount of light on what were once poorly understood connections between Cooper’s storage mechanism and Montague’s quantifying-in scheme. But the type-logical approach does not just rework earlier analyses in a logical light; throughout the rest of this chapter, we demonstrate many ways in which the type-logical approach corrects serious shortcomings with both the quantifying in and storage-based approaches to scoping. We will see examples in which quantifying in and Cooper storage lead to both overgeneration and undergeneration; several known holes in Montague’s and Cooper’s approaches have been patched by the introduction of further rules in the case of undergeneration and by the introduction of several kinds of filters in the case of overgeneration. The resulting analyses have neither the elegance nor the empirical adequacy of the type-logical approach. Not surprisingly, all of these shortcomings can be traced to the point at which Cooper storage and quantifying in break with the type-logical

perspective.

Moortgat (1990a) introduced a binary connective to capture the behavior of quantifiers and other expressions which remain *in situ* syntactically while taking wider scope semantically. The use of such a connective is in keeping with the deductive tradition in type-logical grammars, in which category constructors are construed logically. Moortgat sought a logic in which the category $B \uparrow A$ be assigned to expressions which act locally as B s, but take their semantic scope over an embedding expression of category A . For instance, a generalized quantifier will be given category $np \uparrow s$ because it acts like a noun phrase *in situ*, but scopes semantically at an embedding sentence. We will assume this interpretation of the scoping constructor, as further refined below.

Definition 1 (Scoping Constructor) *The scoping constructor \uparrow is such that:*

- a. $B \uparrow A \in \mathbf{Cat}$, if $A, B \in \mathbf{Cat}$
- b. $\mathit{Typ}(B \uparrow A) = (\mathit{Typ}(B) \rightarrow \mathit{Typ}(A)) \rightarrow \mathit{Typ}(A)$

We will assume that the scoping connective is not associative and binds more tightly than the slashes. The type assignment for such a scoping category is rather subtle. Intuitively, an expression of category $B \uparrow A$ acts on an expression of category A in which it occurs as a B to produce an expression of category A . Thus the result should be the type of A , while the argument should be the type of an A with a B missing, namely $\mathit{Typ}(B) \rightarrow \mathit{Typ}(A)$. Note that for generalized quantifiers, we have the following.

$$(15) \mathit{Typ}(np \uparrow s) = (\mathit{Typ}(np) \rightarrow \mathit{Typ}(s)) \rightarrow \mathit{Typ}(s) = (\mathbf{Ind} \rightarrow \mathbf{Bool}) \rightarrow \mathbf{Bool}$$

Thus our generalized quantifiers every, no, and some are constants of the type $\mathit{Typ}(np \uparrow s)$.

Now we turn to the logical rules for Moortgat's scoping constructor. But there is a problem with the interaction of associativity in the Lambek calculus and the need to mark locations of *in-situ* binders. Unfortunately, no way has been found to provide a complete proof theory for the intended interpretation of a general *in-situ* scoping constructor. Morrill (1994) presents an alternative notion of quantification in a more refined logical setting, for which there is a complete proof theory, but we will not consider Morrill's approach here. We will be content to present Moortgat's proof theory and defer discussion of completeness until Section 9.4. We begin with the sequent form of the rule of use.

$$\frac{\frac{}{x: np, \text{run}: np \setminus s \Rightarrow \text{run}(x): s} D \quad \frac{}{\text{every}(\lambda x. \text{run}(x)): s \Rightarrow \text{every}(\lambda x. \text{run}(x)): s} I}{\text{every}: np \uparrow s, \text{run}: np \setminus s \Rightarrow \text{every}(\lambda x. \text{run}(x)): s} \uparrow L$$

Figure 7.4
Analysis of *everyone ran*

Definition 2 (Scoping Sequent Scheme) *The scoping sequent scheme is:*

$$\frac{\Delta_1, x: B, \Delta_2 \Rightarrow \beta: A \quad \Gamma_1, \alpha(\lambda x. \beta): A, \Gamma_2 \Rightarrow \gamma: C \quad [x \text{ fresh}]}{\Gamma_1, \Delta_1, \alpha: B \uparrow A, \Delta_2, \Gamma_2 \Rightarrow \gamma: C} \uparrow L$$

As with our other schemes that involve hypothetical reasoning, we require the scoping scheme to choose fresh variables to avoid unwanted binding. This rule is most easily understood as involving two stages. The quantifier $\alpha: B \uparrow A$ begins in the context of Δ_1, Δ_2 , and it is over the material in Δ_1, Δ_2 that the quantifier scopes. First, we must establish the subgoal $\Delta_1, x: B, \Delta_2 \Rightarrow \beta: A$, where we substitute a hypothetical $x: B$ in the position originally occupied by the scope-taking element $B \uparrow A$. Note that this is the same subderivation as would allow us to establish that $\Delta_1, \Delta_2 \Rightarrow \lambda x. \beta: A \uparrow B$. This provides the deep connection between the right rule for \uparrow and the left rule for \uparrow , as was first pointed out by Moortgat (1991). In the second stage of the use of the quantifier, the entire context $\Delta_1, \alpha: B \uparrow A, \Delta_2$ is replaced by $\alpha(\lambda x. \beta): A$. This latter term is the result of applying the quantifier's semantic term to the result of the first subderivation with the hypothetical variable abstracted. Intuitively, the quantifier takes its scope at the level of the derivation of the A from the Δ s and the hypothetical B . The A is then used in the context between Γ_1 and Γ_2 to provide the final result. This latter substitution embodies the possibilities for employing the now quantified A in a further derivation; this also indicates how cuts can be incrementally eliminated from derivations involving $\uparrow L$.

The simplest illustration of this rule in action can be found in Figure 7.4. Note that we have used the generalized quantifiers *every* and *some* as instances of the category $np \uparrow s$. In the derivation in Figure 7.4, Γ_1, Γ_2 , and Δ_1 are empty, and $\Delta_2 = \text{run}: np \setminus s$. Given our definition of *every*, an utterance of *everyone ran* will be true if and only if the property denoted by $\lambda x. \text{run}(x)$, which by η -reduction is equivalent to *run*, holds of every individual in the model.

In applying $\uparrow L$, it is often the case that Γ_1 and Γ_2 are empty, and the

$$\begin{array}{c}
\frac{}{D} \\
\frac{x: np, \mathbf{break}: np \backslash s / np, y: np \Rightarrow \mathbf{break}(y)(x): s}{D} \\
\frac{x: np, \mathbf{break}: np \backslash s / np, \mathbf{every}: np \uparrow s \Rightarrow \mathbf{every}(\lambda y. \mathbf{break}(y)(x)): s}{D} \\
\mathbf{some}: np \uparrow s, \mathbf{break}: np \backslash s / np, \mathbf{every}: np \uparrow s \Rightarrow \mathbf{some}(\lambda x. \mathbf{every}(\lambda y. \mathbf{break}(y)(x))): s
\end{array}$$

$$\begin{array}{c}
\frac{}{D} \\
\frac{x: np, \mathbf{break}: np \backslash s / np, y: np \Rightarrow \mathbf{break}(y)(x): s}{D} \\
\frac{\mathbf{some}: np \uparrow s, \mathbf{break}: np \backslash s / np, y: np \Rightarrow \mathbf{some}(\lambda x. \mathbf{break}(y)(x)): s}{D} \\
\mathbf{some}: np \uparrow s, \mathbf{break}: np \backslash s / np, \mathbf{every}: np \uparrow s \Rightarrow \mathbf{every}(\lambda y. \mathbf{some}(\lambda x. \mathbf{break}(y)(x))): s
\end{array}$$

Figure 7.5
Analysis of Simple Scope Ambiguity

second subderivation is carried out by the identity axiom, setting the result $\gamma: C = \alpha(\lambda x. \beta): A$. In this case, we have the following derived inference scheme.

$$(16) \quad \frac{\Delta_1, x: B, \Delta_2 \Rightarrow \beta: A}{\Delta_1, \alpha: B \uparrow A, \Delta_2 \Rightarrow \alpha(\lambda x. \beta): A} D$$

In fact, with cut, the derived scheme in (16) provides the same power as $\uparrow L$. In addition, the derived instance of scoping in (16) illustrates most clearly the parallels between using $np \uparrow s$ in Moortgat's system and quantifying in in Montague's grammar. One pleasant difference is that in the categorial setting, the pronominal lexical entries are not needed for marking the binding point of a quantifier. Instead, the form of the hypothetical antecedent takes care of that detail directly by replacing the quantifier with a variable of the appropriate category in the antecedent. By providing a proper sequent-based logic, we provide an explanation for the form of Montague's quantifying-in scheme.

Just as with quantifying in, the rule of use for the scoping constructor allows for the derivation of relative scope ambiguities. For instance, consider the derivations in Figure 7.5, which make use of the derived scoping scheme in (16).

Three additional benefits of the type-logical approach will soon be apparent. First, the compositional form of the scope constructor in terms of a local category and a scoping category allows a natural generalization to other scoping phenomena. In the next two chapters, we provide accounts of reflexives, reciprocals, pied-piped relative pronouns, and the plural operations of distribution and collection. Later, we will see that

quantificational adverbs can be treated along the same lines. Second, the scoping constructor provides a natural explanation of type raising, both from the perspective of its logic, and its range of application. As we will see in the next section, type raising is nothing more than the rule of proof for quantifiers. With type raising realized logically, we do not need to follow the path of Montague in generalizing lexical entries to the worst possible case. Instead, we will be able to take simple lexical entries and derive their type-lifted forms automatically. Third, the logical basis of quantification provides a principled explanation of the interaction of quantification with other aspects of our grammar. This synergy between the slash constructors, the extraction constructors, co-ordination, and other quantifiers will be the topic of the rest of this chapter. Furthermore, we will see how the constructor-based approach to quantifiers allows a natural treatment of quantifiers occurring in free relative and possessive constructions. In later chapters, we will see how quantifiers interact in the proper fashion with intensional verbs, including intensional transitive verbs, as well as sentence embedding propositional attitude verbs and their control-based and clefted variants.

As with our previous analyses, it is most straightforward to work within the natural deduction version of the scoping scheme.

Definition 3 (Scoping Natural Deduction Scheme) *The natural deduction scheme for scoping is:*

$$\begin{array}{c}
 \vdots \qquad \vdots \qquad \vdots \\
 \vdots \quad \frac{\alpha: B \uparrow A}{x: B} \uparrow E^n \quad \vdots \\
 \vdots \quad \frac{x: B}{\beta: A} \quad \vdots \\
 \vdots \qquad \vdots \qquad \vdots \\
 \hline
 \beta: A \\
 \hline
 \alpha(\lambda x. \beta): A \quad n
 \end{array}
 \qquad [x \text{ fresh}]$$

The natural deduction scheme makes clear the way in which we treat $B \uparrow A$ locally as a B in the derivation of an A . Furthermore, these two stages of the derivation are co-indexed in the natural deduction scheme. It is important to note that the $\uparrow E$ scheme is intended to be read analogously to the sequent presentation in terms of scope. More precisely, it is intended to combine the following two independent derivations in the production of the final result.

$$(17) \frac{\begin{array}{ccc} \vdots & x: B & \vdots \\ \vdots & \vdots & \vdots \end{array} \quad \begin{array}{c} \vdots \\ \alpha: B \uparrow A \end{array}}{\beta: A}$$

In particular, none of the assumptions in the derivation of $\alpha: B \uparrow A$ are available for discharge in the derivation of $\beta: A$ from $x: B$ and the surrounding context. In Section 7.5, we return to this issue and see why this structural understanding of the scope elimination scheme is crucial in preventing some of the unpleasant consequences of Cooper's nonlogical, phrase-structure based storage mechanism.

We can also gain insight into the behavior of the natural deduction scoping scheme by considering its translation into sequent notation.

$$(18) \frac{\Gamma \Rightarrow \alpha: B \uparrow A \quad \Delta_1, x: B, \Delta_2 \Rightarrow \beta: A}{\Delta_1, \Gamma, \Delta_2 \Rightarrow \alpha(\lambda x. \beta): A}$$

This makes clear the scope of the hypothetical assumption, and how the hypothetical variable is bound, as illustrated in (17).

The natural deduction schemes mimic the sequent rules in their effect, as can be seen in the alternative analysis of simple subject-object scope ambiguity, in Figure 7.6.

We have assumed that generalized quantifiers have the expected lexical entries.

$$(19) \text{ a. } \textit{everyone} \Rightarrow \textit{every}: np \uparrow s$$

$$\text{ b. } \textit{someone} \Rightarrow \textit{some}: np \uparrow s$$

We make the standard categorial assumption that generalized determiners subcategorize for their nouns.

$$(20) \text{ a. } \textit{every} \Rightarrow \textit{every}^2: np \uparrow s / n$$

$$\text{ b. } \textit{some} \Rightarrow \textit{some}^2: np \uparrow s / n$$

$$\text{ c. } \textit{no} \Rightarrow \textit{no}^2: np \uparrow s / n$$

Such an assumption follows the semantics in that determiners are relations between the property introduced by their nominal restriction and that introduced by their scope. A derivation involving a generalized determiner can be found in Figure 7.7. Note that we have η -reduced at the quantifier elimination step, simplifying $\lambda x. \textit{bark}(x)$ to \textit{bark} . The bracketing in the semantic result on the root is as expected. Assuming that *white* is an intersective adjective, so that

$$\begin{array}{c}
\frac{\text{someone}_{Lx}}{\text{some}: np \uparrow s} \uparrow E^0 \quad \frac{\text{breaks}_{Lx}}{\text{break}: np \setminus s / np} \quad \frac{\text{everything}_{Lx}}{\text{every}: np \uparrow s} \uparrow E^3 \\
\frac{x: np}{\text{break}(y)(x): s} D \\
\text{every}(\lambda y. \text{break}(y)(x)): s \quad 3 \\
\text{some}(\lambda x. \text{every}(\lambda y. \text{break}(y)(x))): s \quad 0 \\
\hline
\frac{\text{someone}_{Lx}}{\text{some}: np \uparrow s} \uparrow E^2 \quad \frac{\text{breaks}_{Lx}}{\text{break}: np \setminus s / np} \quad \frac{\text{everything}_{Lx}}{\text{every}: np \uparrow s} \uparrow E^0 \\
\frac{w: np}{\text{break}(x)(w): s} D \\
\text{some}(\lambda w. \text{break}(x)(w)): s \quad 2 \\
\text{every}(\lambda x. \text{some}(\lambda w. \text{break}(x)(w))): s \quad 0
\end{array}$$

Figure 7.6
Analyses of *someone breaks everything*

$$\begin{array}{c}
\frac{\text{every}_{Lx}}{\text{every}^2: np \uparrow s / n} \quad \frac{\text{white}_{Lx}}{\text{white}: n / n} \quad \frac{\text{dog}_{Lx}}{\text{dog}: n} / E \quad \frac{\text{barks}_{Lx}}{\text{bark}: np \setminus s} \\
\frac{\text{white}(\text{dog}): n}{\text{every}^2(\text{white}(\text{dog}))}: np \uparrow s \uparrow E^0 \\
\frac{x: np}{\text{bark}(x): s} \setminus E \\
\text{every}^2(\text{white}(\text{dog}))(\text{bark}): s \quad 0
\end{array}$$

Figure 7.7
Analysis of *every white dog barks*

$\text{white} \stackrel{\text{def}}{=} \lambda P. \lambda x. (\text{white}_p(x) \wedge P(x))$, we generate the following analysis.

(21) *Every white dog barks* $\Rightarrow \text{every}^2(\lambda x. \text{white}_p(x) \wedge \text{dog}(x))(\text{bark}): s$

Given our interpretation of the generalized determiner every^2 , an utterance of *every white dog barks* will be true if and only if for every object x , if x is white and a dog, then x barks. In the remaining derivations, we will usually drop the superscript on generalized determiners when their type is clear from context.

The analogy between our elimination scheme for quantifiers and the

operation of Cooper storage should be evident. The point at which elimination is effected corresponds to the point in Cooper's analysis in which a quantifier is replaced by a type-raised variable (Cooper's introduction of a variable type raised to a quantifier rather than just the variable will be a consequence of the type-raising scheme we introduce in the next section.) The point at which the quantifier is retrieved from storage corresponds to the stage at which a quantifier's hypothetical assumption is discharged and it applies semantically. Like Montague, we have a "syntactic" reflection of quantifier scoping, if we care to think of our various proof theories as syntactic theories in the traditional sense. The simple existence of two equivalent presentations with radically different structure demonstrates that a traditional syntactic presentation is unimportant in achieving the correct expression/meaning relation.

7.4 Type Raising and Quantifier Coordination

In this section, we turn our attention to a rule of proof for the scoping constructor. We have already seen Montague's solution to the discrepancy between the natural types assigned to names and to quantifiers, namely individual and generalized quantifier types. As in many other cases, Montague generalized to the worst case by lexically raising names and pronouns up to the type of generalized quantifiers. Lexically, the name *Jo* would be assigned to the category $\lambda P.P(j):gq$. Because quantifiers are assigned to boolean categories, this allowed Montague to provide a grammar for the boolean coordination of arbitrary noun phrases, such as those in the following examples.

- (22) a. [Jo] or [some tall kid] ran.
 b. Francis likes [Felix] and [every dog].

In these cases, we have coordination of a proper name and a quantified noun phrase.

From our logical perspective, type raising is a reasonable rule of proof for the scoping constructor because it is sound with respect to our intuitive characterization of $B \uparrow A$. By recognizing it as such, we avoid the ad hoc lexical character of Montague's use of lexical type raising, which turned out to be rather fragile and ill-suited to generalization. Furthermore, type raising allows us to generalize to the best case lexically, rather than the worst. As pointed out by H. Hendriks (1993), the worst case is often much worse than Montague envisaged when con-

structuring his PTQ fragment. Instead of lexically type raising whenever a higher type might be necessary, we will provide lexical entries of the lowest natural semantic type and allow the grammar to generate other categorizations freely according to the logic of the connectives. We are thus following the spirit of Hendriks' (1993) polymorphic approach to typing. Under Hendriks' approach, lexical entries of the simplest appropriate semantic type are given and then related to infinitely many other types via three schemes of type shifting, the semantic portion of which we provide as follows.

(23) a. Value Raising

$$\frac{\alpha : \sigma_1 \rightarrow \dots \rightarrow \sigma_n \rightarrow \mathbf{Ind}}{\lambda x_1 \dots \lambda x_n . \lambda P.P(\alpha(x_1) \dots (x_n)) : \sigma_1 \rightarrow \dots \rightarrow \sigma_n \rightarrow (\mathbf{Ind} \rightarrow \mathbf{Bool}) \rightarrow \mathbf{Bool}} \text{VR}$$

b. Argument Raising

$$\frac{\alpha : \sigma_1 \rightarrow \dots \rightarrow \sigma_{n-1} \rightarrow \mathbf{Ind} \rightarrow \sigma_{n+1} \rightarrow \dots \rightarrow \sigma_m \rightarrow \mathbf{Bool}}{\lambda x_1 \dots \lambda x_{n-1} \lambda Q \lambda x_{n+1} \dots \lambda x_m . Q(\lambda x_n . \alpha(x_1) \dots (x_m)) : \sigma_1 \rightarrow \dots \rightarrow \sigma_{n-1} \rightarrow ((\mathbf{Ind} \rightarrow \mathbf{Bool}) \rightarrow \mathbf{Bool}) \rightarrow \sigma_{n+1} \rightarrow \dots \rightarrow \sigma_m \rightarrow \mathbf{Bool}} \text{AR}$$

c. Argument Lowering

$$\frac{\alpha : \sigma_1 \rightarrow \dots \rightarrow \sigma_{n-1} \rightarrow ((\mathbf{Ind} \rightarrow \mathbf{Bool}) \rightarrow \mathbf{Bool}) \rightarrow \sigma_{n+1} \rightarrow \dots \rightarrow \sigma_m \rightarrow \mathbf{Bool}}{\lambda x_1 \dots \lambda x_{n-1} \lambda x_n \lambda x_{n+1} \dots \lambda x_m . \alpha(x_1) \dots (x_{n-1})(\lambda P.P(x_n))(x_{n+1}) \dots (x_m) : \sigma_1 \rightarrow \dots \rightarrow \sigma_{n-1} \rightarrow \mathbf{Ind} \rightarrow \sigma_{n+1} \rightarrow \dots \rightarrow \sigma_m \rightarrow \mathbf{Bool}} \text{AL}$$

The first operation involved is *value raising*, which raises a category that produces an individual as a final result to one that produces the corresponding quantifier as a result. This is computed by simply distributing ordinary type raising through all of the arguments. The second operation is *argument raising*, and it involves replacing an individual argument to a function that produces a boolean result and applies the quantifier semantically by distributing through the arguments and binding the variable that is being replaced. The third operation, *argument lowering*, replaces a quantifier argument in a function that produces a boolean result with an individual argument. The semantics is calculating by the result of applying the original function to the type-raised version of the individual. Assuming that the individual results are noun phrases and the sentential results are sentences, all of these operations are derivable using our quantificational schemes; we leave this to Exercise (7-9).

Hendriks motivates many more instances of these schemes than were applied lexically by Montague, and we consider many of his examples below. Fortunately, we do not need to postulate Hendriks' type-shifting

$$\frac{\frac{}{\mathbf{j}: np \Rightarrow \mathbf{j}: np} I}{\mathbf{j}: np \Rightarrow \lambda V.V(\mathbf{j}): np \uparrow s} \uparrow R$$

Figure 7.8
Noun Phrase Type Raising

operations directly. The relevant instances are all derivable using the schemes for the slashes and the quantifiers (see Exercise (23)). The approach we describe also differs from Hendriks' in that we maintain a single type for each syntactic category, whereas Hendriks associates each category with the (potentially infinite set of) types to which it can shift. Moortgat's introduction of the scoping constructor was, in fact, originally motivated by the desire to capture Hendriks' approach to quantification in a strict type-logical setting. As it turns out, Hendriks' semantic type shifts are all derivable in our approach from the simple quantifier rules of use and proof, when combined with Lambek's existing rules for the slashes (see Exercise (7-9)). Thus one way of viewing our approach is as providing a type-logical basis for the collection of operations proposed by Hendriks.

The sequent form of the rule of proof we will employ for the scoping constructor is as follows.

Definition 4 (Scoping Sequent Scheme) *The scope sequent scheme is:*

$$\frac{\Gamma \Rightarrow \alpha: A}{\Gamma \Rightarrow \lambda x.x(\alpha): A \uparrow B} \uparrow R \quad [x \text{ fresh}]$$

Note that as with our other rules introducing variables, we must choose fresh ones for scope introduction. The simplest instance of this scheme allows us to produce generalized quantifiers from noun phrases, as shown in Figure 7.8.

Before considering further applications of the rule of proof for the scope constructor, we provide it in natural deduction format for ease of use.

Definition 5 (Scoping Introduction Natural Deduction) *The scope introduction natural deduction scheme is:*

$$\frac{\begin{array}{c} \vdots \\ \alpha: A \end{array}}{\lambda x.x(\alpha): A \uparrow B} \uparrow I \quad [x \text{ fresh}]$$

$$\begin{array}{c}
\frac{Jo}{j: np} \text{Lx} \quad \frac{and}{and: coor} \text{Lx} \quad \frac{every\ kid}{every(kid): np\uparrow s} D \quad \frac{ran}{run: np\backslash s} \text{Lx} \\
\hline
\frac{\lambda V.V(j): np\uparrow s}{\lambda y.y(j) \wedge every(kid)(y): np\uparrow s} \uparrow I \\
\hline
\frac{x: np}{run(x): s} \backslash E \\
\hline
run(j) \wedge every(kid)(run): s \quad 0
\end{array}$$

Figure 7.9
Analysis of *Jo and every kid ran*

We can now present a derivation of coordination of a noun phrase with a generalized quantifier. The derivation for a very simple example is given in Figure 7.9. Note that we have raised the $j: np$ category for *Jo* to a generalized quantifier, as in our sequent analysis. We have also performed relevant β -reductions, most notably in the application of the coordination. The reason this derivation goes through is that the generalized quantifier category, $np\uparrow s$, is boolean, and thus allowed to participate in coordination, with the right results. We should also note that this scheme will allow us to coordinate two proper names, as in the following examples.

- (24) a. [Jo] and [Brett] ran yesterday.
b. The teacher likes [Jo] but not [Brett].
c. The painting of [New York] or [Chicago] was stolen.

It is sufficient to type raise both names to generalized quantifier categories and then scope the quantifiers as usual. For instance, *Jo and Brett* can be analyzed as $\lambda V.V(j) \wedge V(b): np\uparrow s$.

In general, it is always possible to raise a category using $\uparrow I$ and then eliminate it using $\uparrow E$ to derive the same result. With our slash connectives, we saw that we could eliminate this kind of grammatical spurious ambiguity. As with the slashes, we can simplify proofs involving spurious type raising. For quantifiers, we are not guaranteed to be left with normal terms, though. This is one manifestation of the lack of generality of the rule of proof for quantifiers, as we discuss toward the end of (26). For consistency, we still refer to the reduction operation as normalization.

Definition 6 (Scoping Normalization) *The normalization operation for the scope constructor is:*

$$\begin{array}{c}
 \vdots \quad \vdots \quad \vdots \\
 \vdots \quad \frac{\alpha: A}{\vdots} \quad \vdots \\
 \vdots \quad \frac{\lambda P.P(\alpha): A \uparrow B}{\uparrow E^n} \quad \vdots \\
 \vdots \quad \frac{x: A}{\vdots} \quad \vdots \\
 \vdots \quad \vdots \quad \vdots \\
 \hline
 \beta: B \\
 \hline
 (\lambda P.P(\alpha))(\lambda x.\beta): B
 \end{array}
 \quad \rightsquigarrow \quad
 \begin{array}{c}
 \vdots \quad \vdots \quad \vdots \\
 \vdots \quad \frac{\alpha: A}{\vdots} \quad \vdots \\
 \vdots \quad \vdots \quad \vdots \\
 \hline
 \beta[x \mapsto \alpha]: B
 \end{array}$$

Note that the resulting terms are equivalent by two applications of β -reduction.

We will see below that the type-raising scheme derived by the scope introduction scheme has other uses, in addition to type raising names to allow them to coordinate with quantifiers. In particular, type raising will allow the proper interactions of quantifiers with expressions taking quantified arguments. Such expressions will include some intensional verbs, which take quantified subjects, and cases of auxiliaries, negation, and raising verbs, all of which take a quantifier as a controller. Furthermore, the same action of type raising lies at the root of certain other coordination phenomena, including some especially puzzling cases involving plurals.

7.5 Embedded Quantifiers

In this section, we concentrate on occurrences of quantifiers embedded within other quantifiers. In particular, we consider quantifiers occurring as prepositional complements, as nominal complements, and within relative clauses. We first consider the case of quantifiers occurring as objects of prepositional phrases, as illustrated in the following examples.

- (25) a. Every kid in some class studied.
 b. Some kid in no class slept.

These sentences display quantifier scope ambiguity. For instance, the first sentence, (25)a has a reading in which for some particular class, every kid in that class studied. It has a second reading which requires

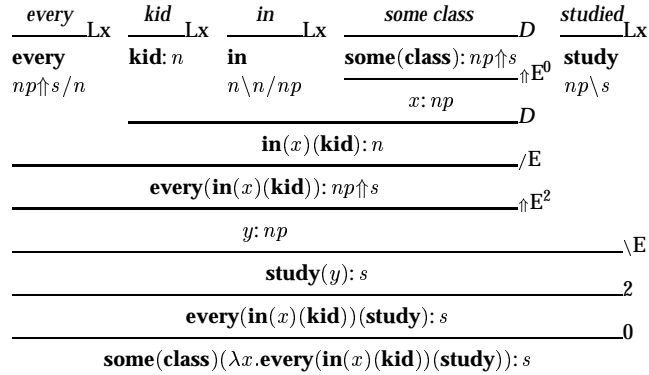


Figure 7.10
 Analysis of *every kid in some class studied*

every kid who happened to be in some class or other to have studied. The second sentence, (25)b is analogous, although for some reason, negative or downward entailing quantifiers seem much harder to scope widely than upward entailing ones. Thus the reading in which there is no class such that a some kid in that class slept is rather difficult to get without strong stress on *no class*. With the lexical entries we have provided for prepositions, which take noun phrase complements, we are only able to derive the first of these readings, which we provide in Figure 7.10.

Cooper’s storage mechanism encounters a subtle problem when faced with embedded quantifiers. For instance, under any reasonable category assignment to nominal modifying prepositions, the possibility for storage will generate the following kind of derivation.

$$(26) \text{ every kid in some class passed} \Rightarrow \text{pass}(x); x/\text{every}(\text{in}(y)(\text{kid})), y/\text{some}(\text{class}): s$$

This derivation is problematic because Cooper’s scoping closure condition, (13)b, entails the existence of both of the following scope-resolved meanings, only the first of which is desirable.

- (27) a. $\text{some}(\text{class})(\lambda y. \text{every}(\text{in}(y)(\text{kid}))(\lambda x. \text{pass}(x)))$
- b. $* \text{every}(\text{in}(y)(\text{kid}))(\lambda x. \text{some}(\text{class})(\lambda y. \text{pass}(x)))$

The second derivation produces an unwanted free variable by scoping the quantifiers in non-nested order.

To circumvent this problem, Cooper (1979) restricted storage to only apply to quantifier meanings with an empty store. That is, Q is required to be the empty string in the storage closure condition that requires $\lambda P.P(x); x/\alpha \cdot Q$ to be in a quantifier's meaning set if $\alpha; Q$ is. Thus the subject will be assigned a meaning set containing $\text{every}(\text{in}(y)(\text{kid})); y/\text{some}(\text{class})$, but not $\lambda P.P(x); x/\text{every}(\text{in}(y)(\text{kid})), y/\text{some}(\text{class})$. It is this latter category which contributes to the undesirable free variable in (27)b. Coincidentally, this restriction of storage to quantifiers with empty stores also prevents a quantifier that has already been stored from being stored again. This prevents a great deal of spurious ambiguity that is eliminated in our system by means of derivation normalization. That is, we might find $\lambda P.P(x); x/\alpha$ in a meaning set, without finding the redundant doubly quantified variant $\lambda R.R(y); y/\lambda P.P(x); x/\alpha$. Of course, our logic deals with the spurious ambiguity of type raising and then quantifying in by cut-elimination in the sequent setting, and by normalization in the natural deduction setting. Unfortunately, Cooper's restriction on storage is not adequate. Although it prevents readings with unbound variables, it also blocks readings in which a quantifier and a quantifier nested within it outscope some other quantifier. For instance, this would arise in Cooper's grammars in the case of an object quantifier and a quantifier nested in the object not being allowed to outscope the subject. We should be able to derive the following.

$$(28) \text{ someone likes every toy in some store} \Rightarrow \\ \text{some}(\text{store})(\lambda x.\text{every}(\text{in}(x)(\text{toy}))(\lambda y.\text{some}(\lambda z.\text{like}(y)(z))))$$

But this requires both the quantifiers from the object and the embedded prepositional object to be stored simultaneously. Of course, the sequent in (28) is derivable in our system, just as it is in Cooper's system without the added restriction.

From the sequent presentation of our quantifier logic, it is clear that unbound variables cannot arise. In a derivation of $\Gamma_1, Q: np \uparrow s, \Gamma_2 \Rightarrow Q(\lambda x.\alpha): s$ from a derivation of $\Gamma_1, x: np, \Gamma_2 \Rightarrow \alpha: s$, any free occurrences of x in α become bound in the result. Furthermore, the parallel between sequent derivations and natural deduction ones, as indicated in (17), ensures that the same property holds. The assumption $x: B$, which introduces the free variable x , can only be used in the derivation of $\beta: A$, at which point the assumption is discharged and all free occurrences of x in β are bound. Because of the similarity of our natural deduction char-

$$\begin{array}{c}
\frac{\text{every}}{\text{every}: np \uparrow s / n} \text{Lx} \quad \frac{\text{kid}}{\text{kid}: n} \text{Lx} \quad \frac{\text{in}}{\text{in}: n \setminus n / np} \text{Lx} \quad \frac{\text{some class}}{\text{some(class): } np \uparrow s} D \quad \frac{\text{ran}}{\text{ran}: np \setminus s} \text{Lx} \\
\frac{\text{some(class): } np \uparrow s}{\uparrow E^0} \\
\frac{y: np}{D} \\
\frac{\text{in}(y)(\text{kid}): n}{/E} \\
\frac{\text{every}(\text{in}(y)(\text{kid}): np \uparrow s)}{\uparrow E^2} \\
\frac{x: np}{\setminus E} \\
\frac{\text{ran}(x): s}{0} \\
\frac{\text{some(class)}(\lambda y. \text{ran}(x)): s}{2} \\
\text{every}(\text{in}(y)(\text{kid}))(\lambda x. \text{some(class)}(\lambda y. \text{ran}(x))): s
\end{array}$$

Figure 7.11
Non-Analysis Violating Independence

acterization of scoping and Cooper storage, it is tempting to consider potential derivations such as that in Figure 7.11. But the structure in Figure 7.11 is not a derivation because there is simply no way to match all of its rule applications to their respective schemes. The problem is that the subderivations are not independent in the sense required by the definition, as depicted in (17). Specifically, the quantifier elimination step $\uparrow E^2$ is unwarranted because $\text{some(class)}(\lambda y. \text{ran}(x)): s$ is not derived exclusively from the assumption $x: np$ and the remaining context $\text{ran}: np \setminus s$, as would be necessary for $\uparrow E$ to be licensed. Finally, it should be noted that the sequent in (28), which is blocked by Cooper's restriction on storage, is generated by our approach.

From the previous analysis, we see that the essential problem for Cooper's approach is that it does not capture the structural relationship between the point at which a quantifier is stored and the point at which it is scoped. Moortgat's type-logical scheme for quantification, on the other hand, enjoys all of the pleasant properties of Cooper's storage mechanism with none of the unpleasant side effects such as free variables in the case of the first version of the storage condition and undergeneration in the case of the second. The incorporation of further structure into Cooper's storage mechanism was independently proposed by Keller (1988) and Gerdemann and Hinrichs (1990). Their approach was to include an explicit indication of the nesting relations between quantifiers in the store. With such a marking, nesting quantifiers can be forced to apply before quantifiers nested inside them, thus capturing the appropriate scope restrictions. From our point of view,

the maneuvering of Keller and of Gerdemann and Hinrichs can be explained by appeal to the logical structure of quantifier elimination; they have done nothing more than to encode the logical structure of quantifier derivations in a phrase-structure setting. Even though non-logical theories such as Cooper's can occasionally be patched by building-in some of the logical structure, our theory not only avoids such patchwork, but also explains a wide range of further interactions between the logical structure of quantification, complementation, unbounded dependency constructions and coordination. In these cases, many of which we consider in the remainder of this chapter, it is not at all clear how even a logically reformulated version of Cooper storage will be able to generate the correct range of possible readings.

Although we do not generate unwanted derivations with unbound variables, we still only generate one reading for each of the sentences in (25) with nested quantifiers. In the case of (25)b, we should be able to generate the following analysis, where the prepositional object quantifier is scoped *in situ*.

$$(29) \text{ some kid in no class slept} \Rightarrow \\ \text{some}(\lambda x.\text{kid}(x) \wedge \text{no}(\text{class})(\lambda y.\text{in}(y)(x)))(\text{sleep})$$

A popular method for dealing with quantifiers that take scope at sub-sentential units was to provide additional rule schemes allowing quantifiers to scope within other boolean categories. For instance, Montague allowed quantifying in at the level of nouns and verb phrases, a move echoed by Cooper in his generalized scoping schemes. Furthermore, such extensions have been proposed in discourse representation theory grammars (Roberts 1987a), and even in the government-binding theory of logical form (May 1985). The fundamental idea is the same as that of generalized coordination, namely that quantification can be distributed through arguments to boolean types. In a simple Montagovian setting, this would allow the following scheme.

$$(30) \text{ if } e \cdot \text{he}_k \cdot e'' \Rightarrow \phi: n \text{ and } e' \Rightarrow \alpha: gq, \\ \text{then } e \cdot e' \cdot e'' \Rightarrow \lambda y.\alpha(\lambda x_k.\phi(y)): n \quad (\text{Quantifying-In Nominal})$$

The same semantic scheme would allow quantification within verb phrases (recall that nouns and verb phrases are assigned the type of properties).

Unfortunately, Montague's general scheme not only fails to generate the correct result, but further allows the generation of incorrect readings.

For instance, consider the following derivation.

- (31) a. $kid\ in\ he_4 \Rightarrow \mathbf{in}(x_4)(\mathbf{kid}): n$
 b. $kid\ in\ no\ class \Rightarrow \lambda y.\mathbf{no}(\mathbf{class})(\lambda x_4.\mathbf{in}(x_4)(\mathbf{kid})(y)): n$
 $\equiv \lambda y.\mathbf{no}(\mathbf{class})(\lambda x_4.\mathbf{kid}(y) \wedge \mathbf{in}_2(x_4)(y)): n$

The logical equivalence in (31)b follows from our definition of prepositional contents. Now notice that an individual a has the semantic property assigned to *kid in no class* if and only if there is no class b such that a is a kid and a is in b . But this holds trivially if a is not a kid. Of course, the nominal *kid in no class* denotes a property that only holds of kids; the prepositional phrase, even with a quantified object, is restrictive by nature. The problem with the nominal quantifying-in scheme is that the quantifier takes scope over not just the preposition's restrictive content, but also over the restriction supplied by the common noun. Note that this result is not due to our meaning postulates for prepositions; no matter what kind of content is assumed for nominal modifiers, the situation in which an embedded quantifier outscopes the head noun's content will always arise.

Within a type-logical approach, we are not free to postulate arbitrary, non-logical rule schemes. Instead, if we want quantifiers to reduce within nominals, we could simply make the following kind of category assignment.

- (32) $everyone \Rightarrow \lambda P.\lambda x.\mathbf{every}(\lambda z.P(z)(x)): np \uparrow n$

This would mimic Montague's nominal quantifying-in scheme, allowing the derivation of (31)b. But rather than following the errant path of allowing quantifiers to reduce within nominals, we instead isolate the problem in the lexical entry previously assigned to prepositions, as suggested by Cooper (p. c.). Our revised lexical entries for prepositions, along with the meaning postulates reflecting their intersective nature, are exemplified as follows.

- (33) a. $in \Rightarrow \mathbf{in}: n \setminus n / (np \uparrow s)$
 b. $\mathbf{in} \stackrel{\text{def}}{=} \lambda Q.\lambda P.\lambda x.P(x) \wedge Q(\lambda y.\mathbf{in}_2(y)(x))$

We are now in a position to generate a proper analysis of the narrow scope of an embedded quantifier, as shown in Figure 7.12. Substituting the meaning of the preposition into the result, we have the following.

- (34) $\mathbf{in}(\mathbf{no}(\mathbf{class}))(\mathbf{kid}) \equiv \lambda x.\mathbf{kid}(x) \wedge \mathbf{no}(\mathbf{class})(\lambda y.\mathbf{in}_2(y)(x))$

$$\begin{array}{c}
\frac{\text{every}}{np \uparrow s / n} \text{Lx} \quad \frac{\text{kid}}{\text{kid}: n} \text{Lx} \quad \frac{\text{in}}{\text{in}: n \setminus n / (np \uparrow s)} \text{Lx} \quad \frac{\text{no class}}{\text{no(class)}: np \uparrow s} D \quad \frac{\text{smiled}}{\text{smile}: np \setminus s} \text{Lx} \\
\frac{\text{in(no(class))}: n \setminus n}{\text{in(no(class))(kid)}: n} \text{E} \\
\frac{\text{every(in(no(class))(kid))}: np \uparrow s}{y: np} \uparrow \text{E}^0 \\
\frac{\text{smile}(y): s}{\text{every(in(no(class))(kid))(smile)}: s} \text{E} \\
0
\end{array}$$

Figure 7.12
Analysis of *every kid in no class smiled*

$$\begin{array}{c}
\frac{\text{in}}{\lambda Q. \lambda P. \lambda x. P(x) \wedge Q(\lambda y. \text{in}_2(y)(x)): n \setminus n / (np \uparrow s)} \text{Lx} \quad \frac{[z: np]^1}{\lambda R. R(z): np \uparrow s} \uparrow \text{I} \\
\frac{\lambda P. \lambda x. P(x) \wedge \text{in}_2(z)(x): n \setminus n}{\lambda z. \lambda P. \lambda x. P(x) \wedge \text{in}_2(z)(x): n \setminus n / np} \text{E} \\
\text{/I}^1
\end{array}$$

Figure 7.13
Derivation of Standard Preposition Lexical Entry

Although we have raised the object type for prepositions, we have done so for semantic reasons; quantified objects do not scope over the nominal modified by prepositions. It is worth noting that our original lexical entries for prepositions can be easily derived from that in (33a) by combining type raising and abstraction. Such a derivation for the case of *in* is shown in Figure 7.13, with the semantic term for the preposition given in full. Thus lexical entries explicitly seeking a noun phrase argument are no longer needed; they can be derived from the entries seeking quantified objects by hypothesizing a noun phrase and raising it to a quantifier for the sake of the derivation. Analogous derivations may be carried out on any boolean category with a generalized quantifier argument. The general pattern is known as *argument lowering*. The combination of the logic of slashes and of the scoping constructor thus generate all of the instances of argument lowering proposed by H. Hendriks (1987, 1993) as a purely semantic operation (see Exercise (7-9)).

Just as for modifying prepositions, we need to raise the categories assigned to relational nouns and case-marking prepositions.

$$(35) \text{ a. } \textit{picture} \Rightarrow \lambda Q. \lambda x. Q(\lambda y. \textit{picture}(y)(x)): n / (np_{of} \uparrow s)$$

$$\text{b. } of \Rightarrow \lambda Q.Q: np_{of} \uparrow s / (np \uparrow s)$$

Lexical entries of this form allow the following analysis purely by functional application.

$$(36) \text{ picture of every kid} \Rightarrow \lambda x.\text{every}(\text{kid})(\lambda y.\text{picture}(y)(x))$$

Note that these lexical entries are compatible with the assumption that quantifiers should be allowed to reduce within nominals in general. Of course, as with the modifying prepositions, the original lexical entries for these categories can be derived by argument lowering, given the type-raising possibilities for noun phrases. The semantic issues surrounding relational nouns are subtle; for one example of why this is the case, see Exercise (7-28).

The other case of nominal modification that we have seen that might involve quantification arises in relative clauses.

- (37) a. Some kid who entered every event is tired.
 b. Every kid who entered some event received a ribbon.
 c. Jo likes some movie which every student likes.
 d. Every kid who is in some class passes every exam.

If quantifiers were allowed to freely scope out of relative clauses, we would have a large number of readings for these sentences. Although intuitions vary widely, it is usually assumed that relative clauses are so-called *scope islands* in the sense that quantifiers embedded in an island cannot take scope outside of the island (Ross 1967; Rodman 1976). For instance, the sentence (37)a is assumed to unambiguously mean that there is a particular kid, say x , such that x entered every event and is tired. Similarly, (37)b cannot mean that there is a particular event for which every student who entered it received a ribbon, but must mean that every kid who entered any event received a ribbon. Of course, in this second case, it may be a single ribbon which every kid received or a different ribbon for each kid; there is no restriction on subject and object quantifiers scoping with respect to one another.

If we wished to syntactically restrict quantifiers from scoping out of relative clauses, we would need to see how the quantifiers can be reduced within the relative clauses and also how they can be prevented from being reduced outside of the relative clause in which they are found. The first of these issues is, in fact, already resolved. The scoping schemes already interact in the correct way with both the slash schemes and the extraction scheme. In the case of subject relative clauses, we can

- b. *movie which every kid likes* \Rightarrow
 $\lambda x.\text{movie}(x) \wedge \text{every}(\text{kid})(\lambda y.\text{like}(x)(y)): n$

Note that we have not performed the η -reduction of $\lambda y.\text{like}(x)(y)$ to $\text{like}(x)$ as was done in the derivation in Figure 7.15.

While this explains how we can reduce quantifiers within expressions which are not full sentences, it does not explain why in some cases, we cannot extract quantifiers out of relative clauses. Although the data are subtle (see Cooper 1983), the type-logical approach lends itself to a highly elegant characterization of islands. To represent islands, a unary operator is used, whose proof theory is that of a minimal modal logic. Details of such an account are presented in Morrill (1992b; 1994a); a general type-logical account of the logical possibilities for modalities can be found in (Moortgat and Oehrle 1993; Moortgat 1994). In these works, a general solution to islandhood and its interaction with coordination, quantification, and unbounded dependencies is presented. Furthermore, the same kind of modal techniques that allow islands to be represented also show a great deal of promise for the treatment of word order (Moortgat and Oehrle 1994).

7.6 Quantifiers and Coordinate Structures

One of the benefits of our categorial approach to complementation is the ease with which coordinate structures can be generated. Things are no different with the addition of quantifiers. We will be able to generate the appropriate range of readings for a wide range of coordinated sentences with quantifiers. We have already seen that quantifiers themselves can be coordinated, because $np \uparrow s$ is a boolean category.

- (39) a. [Some teacher] and [every student] ran.
 b. Jo hit [every student] or [some teacher].

There is no scope interaction between quantifiers when they are themselves coordinated. For instance, there is no reading of (39)a which has the universal and existential taking scope with respect to one another. But such scope interaction could be derived by first eliminating the quantifiers, and then type raising the hypothetical noun phrases; coordination can be carried out on the raised hypothetical noun phrases and the quantifiers scoped arbitrarily. Of course, this behavior is not desirable, and is blocked by the categorial treatment of islands (Mor-

rill 1992a, 1994a; Moortgat 1994). In general, Morrill's treatment of coordination prevents any kind of binding into a coordinate structure, thus preventing both extraction and quantification. In the remainder of this section, we will continue to point out the locality enforced by coordination.

In the previous section, we saw that there is no obstacle to quantifiers reducing within incomplete boolean constituents, such as verb phrases. Thus we will be able to derive the correct meanings for examples such as the following.

- (40) a. Jo [likes every class] and [hates every assignment].
 b. [Every kid hates] but [every adult likes] a nap.
 c. The kid [in every track event] or [in some field event] ran.

The verb phrases in (40)a can be coordinated after being analyzed in the same way as the verb phrase containing a quantifier was analyzed in Figure 7.14. When the conjunct is a non-standard constituent, as in the *s/np* categories of (40)b, the proper result is achieved in the same way. The analysis of the conjuncts proceeds as in the unbounded dependency analysis in Figure 7.15, but with the /-introduction scheme being used to discharge the hypothesis rather than the ↑-introduction scheme. The coordinate structures in (40)c are simpler in the sense that they are derivable by simple application from the prepositional lexical entries that take quantified objects. In all of the cases in (40), the quantifiers cannot scope outside of their individual conjuncts, as would be expected if coordinate structures are islands to dependencies. On the other hand, nothing will prevent quantifiers from reducing within an incomplete sentence if the missing elements can be hypothesized.

Some interesting cases of scope interaction arise between coordinate structures and quantifiers when the restriction or scope of the quantifier is coordinated.

- (41) a. Every [kid] or [adult] just ran.
 b. Some [vegetarian] and [socialist] ran.
 c. Every kid [ran] or [jumped].
 d. Some kid [ran] and [jumped].
 e. [Jo likes] but [Brett hates] some class.

We claim that the first sentence is ambiguous depending on whether or not everyone who is either a kid or an adult ran, or whether it is enough that every kid ran or every adult ran. For the second reading,

$$\begin{array}{c}
 \frac{[Q_1: np\uparrow s/n]^2 \quad \frac{kid}{Lx}}{kid: n/E} \quad \frac{or}{Lx} \quad \frac{[Q_2: np\uparrow s/n]^4 \quad \frac{adult}{Lx}}{adult: n/E} \\
 \frac{Q_1(\mathbf{kid}): np\uparrow s}{\lambda Q_1.Q_1(\mathbf{kid}): (np\uparrow s/n)\backslash np\uparrow s} \backslash I^2 \quad \frac{Q_2(\mathbf{adult}): np\uparrow s}{\lambda Q_2.Q_2(\mathbf{adult}): (np\uparrow s/n)\backslash np\uparrow s} \backslash I^4 \\
 \frac{\lambda Q_3.\lambda P.Q_3(\mathbf{kid})(P) \vee Q_3(\mathbf{adult})(P): (np\uparrow s/n)\backslash np\uparrow s}{D}
 \end{array}$$

Figure 7.16
 Analysis of *kid or adult*

the disjunction is said to take wide scope over the quantification. The availability of wide scope disjunction has been challenged repeatedly, despite the fact that such usage is felicitous in many contexts. For instance, in the context of a series of races at a company picnic, the wide-scope reading of *or* in (41)a is more felicitous than the narrow reading. Dale Gerdemann (p. c.) pointed out that there’s a Christmas carol with the following two lines, which are obviously meant to be read with wide scope coordinators.

- (42) a. He’s gonna find out who’s naughty and nice.
- b. He knows if you’ve been bad or good.

To evoke wide scope coordination readings in some situations, Partee and Rooth (1984; 1987) placed examples such as (41)a in the context of the continuation *but I don’t know which*, in which the speaker expresses the distributivity of the disjunction through the entire interpretation (although they argued that (41)a was not itself ambiguous in this way). H. Hendriks’ (1993) noted that such ambiguities are often possible if the entire sentence is embedded within a propositional attitude context, say as the complement to a verb like *believe*. Following Hendriks, we allow all of the possibilities to be freely generated. Similar ambiguities are displayed by the remaining examples in (41). The narrowly scoped coordination analysis of (41)a is as usual; the nouns are simply coordinated as boolean categories, and then are free to act as the complement to the quantificational determiner. The wide-scope disjunctive reading, on the other hand, arises by first lifting the nouns to functors over determiners, and then coordinating at that level. Such an analysis is shown in Figure 7.16. With the coordinate structure seeking a generalized determiner to its left and distributing it into both conjuncts, we can carry out the rest of the analysis as shown in Figure 7.17. Analyses along similar lines are available to allow coordinate structures at any depth to be dis-

$$\begin{array}{c}
\frac{\text{every}}{\text{every}: np\uparrow s/n} \text{Lx} \quad \frac{\text{kid or adult}}{\lambda Q_3.\lambda P.Q_3(\mathbf{kid})(P) \vee Q_3(\mathbf{adult})(P)} \text{D} \quad \frac{\text{ran}}{\text{ran}: np\backslash s} \text{Lx} \\
\frac{\lambda P.\text{every}(\mathbf{kid})(P) \vee \text{every}(\mathbf{adult})(P): np\uparrow s}{(np\uparrow s/n)\backslash np\uparrow s} \backslash E \\
\frac{x: np}{\lambda P.\text{every}(\mathbf{kid})(P) \vee \text{every}(\mathbf{adult})(P): np\uparrow s} \uparrow E^0 \\
\frac{\text{run}(x): s}{x: np} \backslash E \\
\frac{\text{every}(\mathbf{kid})(\text{run}) \vee \text{every}(\mathbf{adult})(\text{run}): s}{\text{run}(x): s} 0
\end{array}$$

Figure 7.17
Analysis of *every kid or adult ran*

$$\begin{array}{c}
\frac{\text{some kid}}{\text{some}(\mathbf{kid})} \text{D} \quad \frac{[Q_3: np\uparrow s]^1}{x: np} \uparrow E^3 \quad \frac{\text{ran}}{\text{ran}: np\backslash s} \text{Lx} \quad \frac{\text{and}}{\text{and}} \text{Lx} \quad \frac{\text{jumped}}{\lambda Q_4.Q_4(\mathbf{jump}): (np\uparrow s)\backslash s} \text{D} \\
\frac{\text{run}(x): s}{\text{run}(x): s} \text{E} \quad \text{coord} \\
\frac{Q_3(\text{run}): s}{\text{run}(x): s} 3 \\
\frac{\lambda Q_3.Q_3(\text{run}): (np\uparrow s)\backslash s}{Q_3(\text{run}): s} \backslash I^1 \\
\frac{\lambda Q_2.Q_2(\text{run}) \wedge Q_2(\text{jump}): (np\uparrow s)\backslash s}{\lambda Q_3.Q_3(\text{run}): (np\uparrow s)\backslash s} \text{D} \\
\frac{\text{some}(\mathbf{kid})(\text{run}) \wedge \text{some}(\mathbf{kid})(\text{jump}): s}{\lambda Q_2.Q_2(\text{run}) \wedge Q_2(\text{jump}): (np\uparrow s)\backslash s} \backslash E
\end{array}$$

Figure 7.18
Analysis of *some kid ran and jumped*

tributed to the top level (modulo any island constraints in a theory that is sensitive to them). For instance, the verb phrase coordination in (41)d can occur at type $np\backslash s$, which allows the existential subject to take wide scope. Alternatively, we have the type-raised analysis in Figure 7.18, in which the verb phrases undergo argument raising before being coordinated. Of course, a similar ambiguity can be found in the coordination of a subject and transitive verb, as in (41)e. Again, our grammar allows the coordinate structure to be analyzed as an s/np or as a $s/(np\uparrow s)$, along the same lines as the analysis in Figure 7.18. Further examples of the interactions between coordinate structures and quantifiers can be found in the exercises. We now simply note that examples such as the following can be provided with a range of correct readings.

- (43) a. Every student [wrote some paper] and [read some book].
b. Jo showed [someone every drawing] but [no one every painting].

The relevant fact for our flexible approach is that both relative scopings are available for both conjuncts.

7.7 Quantification and Negation

When quantifiers occur with negative particles, they may take scope either within or outside the negation, as can be seen in the following examples.

- (44) a. Every student didn't study.
 b. Kim didn't pass every test.
 c. Kim didn't pass several tests.

The first example, (44)a, has two readings. When the quantifier takes widest scope, the sentence states that every student is such that they did not study. When the quantifier takes narrow scope, the sentence states that it is not the case that every student studied. The second example, (44)b, has the same ambiguity (although the wide scope quantifier reading is perhaps better expressed with the negative polarity existential quantifier *any* with a narrow scope). With the third example, (44)c, both readings are more evident. This example may mean that there are several tests which Kim did not pass or that it was not the case that Kim passed several tests (we return to plural noun phrases in the next chapter).

Although the sentences in (44) might suggest an analysis in which the negative particle takes scope, this is not the route that we will follow. A strong motivation for resisting such an analysis is that negatives only “scope” with respect to quantifiers. That is, there is no scope ambiguity in expressions such as *probably didn't study*; the modal adverbial *probably* must outscope the negation (we return to modals in Chapter 11). Instead of treating negative elements as scoping, we instead treat them as a kind of raising verb. Specifically, we will assume the following kind of entry for auxiliaries and negative particles (ignoring, for now, the delicate matter of tense and aspect).

- (45) a. *did* $\Rightarrow \lambda V.V : (np \uparrow s) \setminus s / ((np \uparrow s) \setminus s)$
 b. *didn't* $\Rightarrow \lambda V.\lambda Q.\neg V(Q) : (np \uparrow s) \setminus s / ((np \uparrow s) \setminus s)$
 c. *not* $\Rightarrow \lambda V.\lambda Q.\neg V(Q) : (np \uparrow s) \setminus s / ((np \uparrow s) \setminus s)$

Note that the complements of auxiliaries and negatives are not quite verb phrases, but rather something like a verb phrase but taking a

$$\begin{array}{c}
\frac{\text{Brett}}{\mathbf{b}: np} \text{Ix} \quad \frac{\text{didn't}}{\lambda V. \lambda Q_2. \neg V(Q_2)} \text{Ix} \quad \frac{\text{study}}{\lambda Q_3. Q_3(\mathbf{study}): (np \uparrow s) \setminus s} D \\
\frac{\lambda Q_1. Q_1(\mathbf{b}): np \uparrow s}{\lambda Q_1. Q_1(\mathbf{b}): np \uparrow s} \uparrow I \quad \frac{\lambda V. \lambda Q_2. \neg V(Q_2)}{(np \uparrow s) \setminus s / ((np \uparrow s) \setminus s)} \quad \frac{\lambda Q_3. Q_3(\mathbf{study}): (np \uparrow s) \setminus s}{\lambda Q_3. Q_3(\mathbf{study}): (np \uparrow s) \setminus s} / E \\
\frac{\lambda Q_2. \neg Q_2(\mathbf{study}): (np \uparrow s) \setminus s}{\lambda Q_2. \neg Q_2(\mathbf{study}): (np \uparrow s) \setminus s} \setminus E \\
\neg \mathbf{study}(\mathbf{b}): s
\end{array}$$

Figure 7.19
Analysis of *Brett didn't study*

quantified subject instead. For Montague, and in many other theories, the introduction of the categories in (45) would necessitate a wholesale type lifting of all lexical verb phrases, adverbs, and so on, as was done in (Montague 1970a). From our logical vantage point, we can maintain our natural type assignments for verbs, and simply raise their arguments to the appropriate level when necessary. For instance, the subject of a verb phrase like *ran* can be lifted as has already been shown in Figure 7.18. Similarly, we can raise noun phrases to quantifiers to act as subjects in such constructions. Thus an expression like *Brett didn't study* will be analyzed as $\neg \mathbf{study}(\mathbf{b})$, as shown in Figure 7.19.

With the lexical entries we have, we can use type raising and slash introduction at the same time to produce the two readings of (44)a, as shown in Figure 7.20. The narrow scope reading of the quantifier with respect to the negation, corresponding to the first analysis in Figure 7.20, is derived naturally by application. The wide scope reading of the quantifier is achieved by a combination of the quantifier elimination and introduction schemes. This is exactly the same technique that we have used before to allow a quantifier to take wide scope out of a quantified argument position; the quantifier is eliminated and then immediately raised.

A further subtle case is encountered when higher-order modifiers such as *not* are nested.

- (46) a. Everyone didn't not study.
b. Everyone probably didn't study.

The examples in (46) are three-ways ambiguous with respect to quantifier scope. The quantifier can scope wide, between the two operators, or narrow with respect to both operators. The first and last possibility follow the same derivations as in Figure 7.20; narrowest scope follows from application and widest scope by eliminating the subject quantifier and then type raising it before application. The intermediate case also

student is likely due to suppletion by *no student*. Similarly, *Not Jo ran* is more naturally expressed by *Jo didn't run*. Recall the discussion in Section 3.3.2 of completeness and consistency for quantifiers derived from type raising individuals, which explains the synonymy of verb phrase and sentential negation for sentences with subjects which are names. We do see names negated in response to questions such as *who ran?*, which can be naturally answered with *not Sandy*. Names can also be negated in coordinated contexts such as *Francis and not Brett*.

7.8 Quantification and Definite Descriptions

With our current approach to quantification, it remains feasible to maintain our analysis of the definite determiner as being of the category $\iota: np/n$ (recall that ι is the description operator from higher-order logic). This assignment correctly generates the correct range of readings for sentences containing definites and quantifiers.

- (48) a. The teacher scolded the unruly student.
 b. The teacher praised every student.
 c. The teacher in every class lectured.
 d. The collector from Cleveland with every comic book jumped at the deal.

The first example, (48)a, which contains multiple definite determiners, is not ambiguous. In our grammar, it is assigned a unique reading; expressions of category np/n simply do not allow for scope ambiguities. Even though at the np stage, a definite description such as *the teacher* may be raised to a quantifier, that quantifier will not participate in scope alternations. In the second example, (48)b, the quantified object *every student* must be assigned scope. Even so, our grammar generates only one reading, in which the single teacher scolded every one of the students. In the last two examples, (48)c and (48)d, a scope ambiguity arises with respect to whether the quantifier reduced within the nominal or escapes to take sentential scope. These two analyses are shown in Figure 7.22. For the scoping of the quantifier within the prepositional phrase, simple application suffices. The resulting description, after substitution and reduction, is $\iota(\lambda x. \text{teach}(x) \wedge \text{every}(\text{class})(\lambda y. \text{in}_2(y)(x)))$. Such a term refers to the unique individual who is a teacher and who is in every class. The more plausible reading in this case is the second, in which the description is $\iota(\lambda x. \text{teach}(x) \wedge \text{in}_2(y)(x))$. With the variable

are the same as long as there is at least one student. Otherwise, the analysis in which the universal is given wide scope could be trivially satisfied. Similarly, the scope ambiguities in (48)c and (48)d would have their usual analyses.

But matters are quite different when we consider the sentence that originally inspired Russell's analysis.

(52) The king of France is not bald.

If we treat the definite as a quantifier, then the analysis of (52) proceeds as in Figure 7.20, producing the following two results, as indicated in Chapter 3, (46).

(53) a. $\text{the}(\text{king_of_France})(\lambda x. \neg \text{bald}(x))$
 b. $\neg \text{the}(\text{king_of_France})(\text{bald})$

The key point here is that the first analysis, which Russell considered the primary analysis, only the verb phrase is negated and the existence of a king of France is required for the interpretation to be true. Under the second analysis, in which the quantifier takes narrow scope under the negation, if there is no king of France, the sentence is automatically true.

Proponents of both the quantificational and referential approach to definiteness remain, and we leave the issue unresolved. We conclude by noting as we did in Section 3.5.3 that the debate between Strawson and Russell as to whether or not context plays a role in determining definiteness is orthogonal to the issue of whether the definite determiner behaves as a Russellian quantifier or a Strawsonian definite description. Further contextual restrictions on the interpretation of a noun can be encoded under either approach.

7.9 Possessives

In this section, we consider the use of possessive noun phrases, as found in sentences such as the following.

(54) a. Chris stole Jody's hat.
 b. Every kid's toy broke.
 c. Sandy's student's machine is slow.
 d. Kim's student's computer's monitor's image is fuzzy.

$$\begin{array}{c}
 \frac{\text{Chris}}{c: np} \text{Lx} \quad \frac{\text{stole}}{np \backslash s / np} \text{Lx} \quad \frac{\text{Jody}}{j: np} \text{Lx} \quad \frac{\text{'s}}{np \backslash (np / n)} \text{Lx} \quad \frac{\text{hat}}{\text{hat}: n} \text{Lx} \\
 \hline
 \lambda x. \lambda P. \iota (\lambda y. \mathbf{poss}(x)(y) \wedge P(y)) \\
 \hline
 \lambda P. \iota (\lambda y. \mathbf{poss}(j)(y) \wedge P(y)): np / n \\
 \hline
 \iota (\lambda y. \mathbf{poss}(j)(y) \wedge \mathbf{hat}(y)): np \\
 \hline
 \mathbf{steal}(\iota (\lambda y. \mathbf{poss}(j)(y) \wedge \mathbf{hat}(y))): np \backslash s \\
 \hline
 \mathbf{steal}(\iota (\lambda y. \mathbf{poss}(j)(y) \wedge \mathbf{hat}(y)))(c): s
 \end{array}$$

Figure 7.24
Analysis of *Chris stole Jody's hat*

These examples all involve a noun phrase, followed by 's. Morphologically, the 's is often treated as a *clitic*, where a clitic is understood simply as an expression which seems to attach itself to a word phonologically, but has a wider scope semantically. For instance, in (54)b, the subexpression *kid's* is often treated as a unit lexically. Instead of following this route, we will simply finesse the problem of surface representation and treat the 's as a lexical entry (but see (Kraak 1995) for a type-logical analysis of clitic distribution).

$$(55) \text{'s} \Rightarrow \lambda x. \lambda P. \iota (\lambda y. \mathbf{poss}(x)(y) \wedge P(y)): np \backslash (np / n)$$

The first thing to note about this entry is that it treats possession as a definite relation. Such an analysis is appealing semantically, and can also be supported on syntactic grounds; possessive noun phrases can show up where referential noun phrases are required, as in copula complements and as appositive noun phrase modifiers. Second, we have assumed a binary relation **poss** which holds between two objects if one possesses the other in some sense. For instance, the first sentence above is analyzed as in Figure 7.24. Notice that we have chosen the lexical entry so that the combination of the possessive and its noun phrase complement form a determiner-like category, which then applies to a noun in the ordinary way.

Of course, we also get cases of quantifiers showing up in the possessor slot, as in (54)b. The proper readings of these sentences follow naturally on our analysis, the one for (54)b being given in Figure 7.25. Here we see that the analysis of quantification allows each kid to possess a different toy, though the toy must be unique for each kid.

Williams (1982) claimed that the relation between a possessive and the object "possessed" can be arbitrary. For instance, *Francis's car* can either

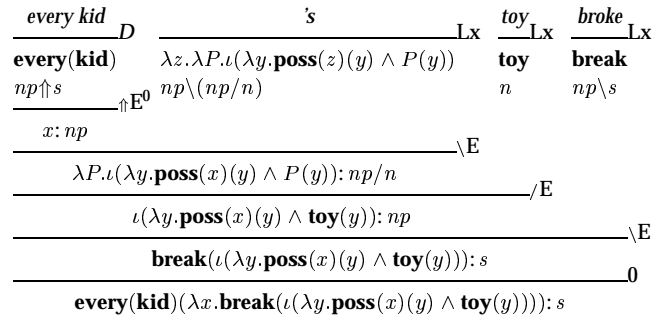


Figure 7.25
 Analysis of *every kid's toy broke*

mean the car Francis owns, the car Francis is standing next to, the one he's renting, and so on. To achieve this effect, we need only interpret the constant **poss** based on contextual factors. In contrast to Williams, Barker (1991) points out some interesting facts concerning the range of interpretations available for the possessive. For instance, consider the following expressions of part-whole relationships.

- (56) a. The table's leg / # The leg's table
- b. The box's cover / # The cover's box
- c. The person's arm / # The arm's person

There is a conventional understanding in utterances of *the X's Y* that *Y* is a part of *X*. As usual, we will focus on the compositional contribution of the possessive rather than its fine-grained lexical semantics. The reader interested in other distributional facts and semantic analyses of the possessive, is urged to consult (Barker 1991).

In English, the possessive construction can also be used in some cases to fill the complement roles of a relational noun.

- (57) a. The sister of Jo / Jo's sister
- b. The picture of Jo / Jo's picture
- c. Jo's picture of Francis / # Jo's sister of Francis
- d. The belief of Francis that Brett would retire
- e. Francis's belief that Brett would retire
- f. # That Brett would retire's belief of Francis

We claim that in the first two cases, (57)a and (57)b, the *of*-marked prepositional object alternates with the possessive with the same effect.

$$\begin{array}{c}
 \frac{Jo}{j: np} \text{Lx} \quad \frac{\text{'s}}{\lambda x. \lambda P. \iota(P(x)): np \setminus (np / (n / np_{of}))} \text{Lx} \quad \frac{picture}{picture: n / np_{of}} \text{Lx} \\
 \hline
 \frac{\lambda P. \iota(P(j)): np / (n / np_{of})}{\iota(\mathbf{picture}(j)): np} \text{E}
 \end{array}$$

Figure 7.26
Analysis of *Jo's picture*

When both appear, as in (57)c, the possessive must be read possessively because the only option for the *of*-phrase is as a complement. Of course, this leads to a pragmatic infelicity in the case of *Jo's sister of Francis*, because sisters aren't typically owned (although consider the situation in which Francis is a puppy with a lot of sisters, each owned by a different individual). The last cases, (57)d–f, indicate that it is only *of*-complements that can be consumed by possessives. To handle such cases, we have two options. First, we could try to generalize the already highly contextualized notion of possession to filling argument roles. Alternatively, we could admit a second lexical categorization for the possessive that consumes a nominal complement which itself sought a complement.

$$(58) \text{'s} \Rightarrow \lambda x. \lambda R. \iota(R(x)): np \setminus (np / (n / np_{of}))$$

An analysis of a noun phrase involving this entry can be found in Figure 7.26. It is also interesting to note that an expression such as *Jo's picture* is ambiguous, due to the detransitivized lexical entry for *picture* (in general, any noun taking a complement also appears as a noun lacking an expressed complement, but with its position bound off semantically by an existential quantifier).

$$(59) picture \Rightarrow \lambda x. some(\lambda y. picture(y)(x)): n$$

This entry allows a purely possessive reading of *Jo's picture*.

$$(60) Jo's picture \Rightarrow \iota(\lambda x. some(\lambda y. picture(y)(x)) \wedge poss(j)(x)): np$$

To further complicate matters, the possessive relation can be indicated by an *of*-marked complement, as long as the object of the preposition is itself possessive.

- (61) a. The picture of {Jo / Jo's / his / him}
 b. The picture of Francis of {his / * him / Jo's / * Jo}

In the first example, (61)a, the full range of complements is allowed, but with the possessives, the relationship is not depiction, but ownership or some other form of possession. Similarly, when an explicit complement for the picture is given, as in (61)b, we get the same pattern as in (57)c. In Exercise (7-11), we consider raising the type of possessives to take quantified complements to deal with expressions such as *every kid's favorite toy* to refer to a single toy.

7.10 Indefinites

The behavior of noun phrases with the indefinite articles *a* and *an* stand in stark contrast both to those with quantifiers and to definite descriptions, although they share properties of both. As a first pass, we might try to analyze *a* as being synonymous with the existential determiner *some*. After all, the following sentences appear to have nearly identical truth conditions.

- (62) a. A student studied.
 b. Some student studied.

But there are at least four ways in which indefinites differ from other quantifiers.

First, indefinites pattern like definites in their ability to appear in contexts demanding so-called *referential noun phrases*, such as the complement to *be* and in *appositive* constructions.

- (63) a. Sandy is {a student / the best student / Terry's friend / no hero / some student / *every student}.
- b. Sandy, {a hero / the best student / *some student / *every student}, is our hardest worker.

These contrasts are perhaps not the strongest evidence, because it is not clear why *no hero* can appear as a complement to *be*, because negatives obviously fail to refer. Also, the pattern of the quantificational determiner *some* is confusing, perhaps because of its close connection to indefinites. There is also a reading with *some student* which can be interpreted as stating that Sandy is an impressive student.

The second point of departure concerns scope islands. Unlike other quantifiers, indefinites can penetrate islands on quantifiers to land in wide scope positions. Consider the difference between the following examples (based on Fodor and Sag 1982; Abusch 1994).

- (64) a. Every student who is in {a / some} class I teach studied hard.
 b. Every teacher overheard the rumor that {a / some} student of mine had been called before the dean.
 c. If {a / some} student in the syntax class cheats on the exam, every professor will be fired.
 d. Each department head believes that it would be damaging for {a / some} professor in her department to quit.

In (64)a, the relative clause forms an island from which ordinary quantifiers such as those with determiners *some* and *every* cannot escape to take wide scope, whereas the indefinite clearly can. Similarly, in (64)b, if *some* occurs, the rumor must be de dicto in the sense that the hearing was about some student or other; with the indefinite *a*, every teacher could have heard a specific rumor about the same student, under a reading where the indefinite takes widest scope. In (64)c, the antecedent clause for the condition forms an island from which a quantified noun phrase cannot escape. Thus the claim is that with *some*, this sentence can only mean that every professor will be fired if any of the students cheat; with an indefinite, there is also a reading whereby there is a particular student such that if that student cheats, every professor will be fired. Example (64)d is similar to the previous three, with the island being an extraposed sentential subject (see Section 11.1.4 for more details on sentential subjects and Section 11.4 for an analysis of extraposition).

A third way in which indefinites differ from other determiners and pattern like definites is in their ability to induce generic readings, as we will see in Section 7.11.

Like quantifiers, indefinites are not required to have scope over the clausal or sentential unit in which they appear. Indefinites can pattern just like existentials in contexts such as the following.

- (65) Everyone in the US drives {some / a} car.

It is apparent that either the indefinite or existential determiners can take narrow scope relative to *everyone* in such cases. This led Fodor and Sag (1982) to claim that indefinites are ambiguous between a reading in which they behave essentially like existential *some*, and a reading in which they behave like demonstrative *that* with the relevant “pointing” required by demonstratives going on in the mind of the speaker rather than in the context (we discuss demonstratives in the context of indexicality in Section 10.3). Such an analysis would generate either the restricted island-bound readings of a quantifier, or would immediately

move to the top level of quantificational structure like other demonstratives. Fodor and Sag's analysis prohibits *intermediate scopings* from being generated. For instance, in (64)b, no reading is generated in which *a student of mine* escapes the relative clause island while staying within the scope of the subject universal, paraphrasable as stating that for each teacher, there was a particular student of mine such that the teacher overheard the rumor that that student had been called before the dean. King (1988) provides the putative counterexample (66)a, claiming that an intermediate scoping is possible. Abusch (1994) agrees, citing the logical independence of the readings in the structurally identical (66)b.

- (66) a. Each author in the room despises every publisher who would not publish a book that was deemed pornographic.
 b. Every professor rewarded every student who read a book he had recommended.

The data is obviously quite subtle here. King's claim is that there is a reading where the book depends on the author, thus escaping the relative island, but scoping within the universal subject. In any case, because we are not particularly concerned with islands here, we will turn to the most compelling reason for differentiating indefinites from existentials.

The fourth, and perhaps most striking way in which indefinites differ from existentials is in their ability to be assimilated into quantifiers and conditionals. Consider the following examples of so-called *donkey sentences*, which were first noted by Geach (1962).

- (67) a. Every farmer who owns a donkey beats it.
 b. If a farmer owns a donkey, he beats it.

The reading of (67)a that concerns us here is the one that is true if and only if every pair consisting of a farmer and a donkey such that the farmer owns the donkey stand in the beating relation. There is a synonymous reading for (67)b; if x is a farmer, y is a donkey, and x owns y , then x beats y . The puzzle that arises is how a basically existential determiner like *a* can behave universally in this way.

Lewis (1975) proposes a two-pronged solution to this problem. First, he assumes that indefinites are represented as free variables with restrictions on their interpretations. Abusch (1994) further develops the role of the restrictions, which we will avoid for the time being. Lewis then proposes a new logical mechanism of *unselective binding* in which

a quantifier can bind all of the variables in its scope. He proposes notations $\forall\phi$ and $\exists\phi$, which he takes to bind all of the variables free in ϕ . This leads to the following definition.

$$(68) \llbracket \forall\phi \rrbracket_{\mathcal{M}}^{\theta} = \begin{cases} \text{yes} & \text{if } \llbracket \phi \rrbracket_{\mathcal{M}}^{\theta'} = \text{yes for all } \theta' \sim_{Free(\phi)} \theta \\ \text{no} & \text{otherwise} \end{cases}$$

Recall that $Free(\phi)$ is the set of free variables in ϕ . The notation $\theta \sim_X \theta'$ is meant to indicate that θ agrees with θ' on every variable not in the set X . Thus in this case, $\forall\phi$ is true under an assignment if ϕ is true under every assignment that at most differs in the assignments to variables free in ϕ . $\exists\phi$ is handled in the same way. Lewis suggests that other operators also exploit unselective binding, mentioning as examples implications and quantificational adverbs (see Section 12.5.8 for a discussion of quantificational adverbs). Lewis also points out that unselective binding does not introduce any new logical power above ordinary universal and existential quantifiers.

Lewis uses unselective binding to represent Geach's example (67)a as follows.

$$(69) \forall((\text{farmer}(x) \wedge \text{donkey}(y) \wedge \text{own}(y)(x)) \rightarrow \text{beat}(y)(x))$$

Such analyses are acceptable from a logical point of view for this case. Without unselective binding, we can approximate these readings by binding sequences of individuals as in the following formula, which is logically equivalent to D. Lewis's (69).

$$(70) \text{every}_{\text{Ind} \times \text{Ind}} (\lambda x. \text{farmer}(\pi_1(x)) \wedge \text{donkey}(\pi_2(x)) \wedge \\ \text{own}(\pi_2(x))(\pi_1(x)) \\ (\lambda x. \text{beat}(\pi_2(x))(\pi_1(x)))$$

Although this logical expression appears to capture the meaning of the donkey sentence (67)a, it is not at all clear how such meanings can be generated in a compositional fashion. There are also logical problems concerning binding from a higher clause; there has to be some mechanism to restrict the variables that unselective quantifiers bind. For instance, there should be a way to refine an expression such as $(\exists y)\forall\phi(x, y, z)$ such that the y is bound by the selective existential and not the unselective universal. This problem was solved by the independent introduction of discourse representation theory (DRT) by Kamp (1981) and file-card semantics by Heim (1982), theories which only differed in their notation. Both theories provide analyses of unselective quantification which allow binding to be more explicitly marked. The

markers for scope are known as *discourse referents*, a construct introduced independently by Karttunen (1976) to address the related issues of presupposition and the potential for inter-sentential anaphora.

In addition to enabling scope to be marked, Heim and Kamp were able to uniformly treat indefinites as introducing discourse referents, which are essentially scope-marked variables with restrictive side conditions. This has allowed compositional syntactic/semantic theories to be developed (Carpenter 1989; Zeevat 1989; Muskens 1993). Discourse representation theories are also adept at handling constraints on intrasentential and intersentential anaphora, a topic to which we return in Chapter 9.

In order to reconcile ideas from discourse representation theory with more standard logical representations and the idea of discourse referents as being updated, Groenendijk and Stokhof (1991) introduced a system they call *dynamic predicate logic* (DPL). Roughly, the point of DPL is to treat the existential quantifier like an assignment statement in a programming language, threading such assignments from left to right through the evaluation of a conjunction. This dramatically changes the conventional notion of quantifier scoping. For instance, it becomes possible to interpret $(\exists x)\phi \wedge \psi$ with the existential quantifier binding the occurrences of x in ψ , even though they are not properly within its scope. The way this is done is by treating the subterm $(\exists x)\phi$ as setting the value of x and passing this setting on to the evaluation of the second conjunct ψ . In classical first-order logic, such changes to the assignment would only last until the subterm over which the quantifier scoped was evaluated. With their dynamic notion of scoping, Groenendijk and Stokhof are able to treat indefinites as introducing existential quantifiers in the ordinary way. By interpreting the existentials differently, this allows the binding of an existential to survive beyond its usual scope limitations. Chierchia (1992) extends Groenendijk and Stokhof's logic to a higher-order intensional setting, and contains a quite detailed discussion of the resulting possibilities for anaphoric binding.

We will not have much more to say concerning the logic of indefinites. Integrating their logic with the logic of other quantifiers remains an interesting open problem (though see Kamp and Reyle 1993; Chierchia 1992).

7.11 Generics

In this section, we discuss the semantics of so-called *generics*. Our presentation closely follows that of Schubert and Pelletier's (1987) survey, and the interested reader is urged to consult that paper for further details. In the null context, the following sentences, drawn from Schubert and Pelletier (1987), will likely be interpreted generically.

- (71) a. Snakes are reptiles.
 b. Telephone books are thick books.
 c. Guppies give live birth.
 d. Italians are good skiers.
 e. Frenchmen eat horsemeat.
 f. Unicorns have one horn.

As Schubert and Pelletier point out, these sentences are made true by different absolute numbers and ratios of instances of their subjects having the property introduced by the verb phrase. For instance, every snake is a reptile, but not every telephone book is a thick book. Only female guppies give live birth, and not all female guppies have that privilege. Although most Italians are not good skiers, there is a higher percentage of good Italian skiers than good skiers of other nationalities. And while few Frenchmen may eat horsemeat, the mere fact that some of them do seems enough to license (71)e. And in the last case, note that there are no unicorns. This seems to prevent any kind of analysis of generics by means of a logical quantifier. Certainly *every* would be too strong, and the latter examples in (71) seem to argue against a weaker version with a quantifier like *most*.

Although the examples in (71) all involve *bare plurals* (see Section 8.2), and are all in the present tense, generics may appear in any tense and with any kind of referential noun phrase. For instance, consider the following variants of (71)a.

- (72) a. Snakes are reptiles.
 b. A snake is a reptile.
 c. The snake is a reptile.

Thus the notion of a generic is not intrinsically linked to that of plurality. To see that the tense can vary for generic sentences, consider the following.

- (73) a. Workers are not protected now.

- b. Workers were not protected last year.
- c. Workers will not be protected next year.

Thus an immediate puzzle that is raised by generics is how the genericity is signaled syntactically and semantically. In the rest of this section, we consider the most well known approaches that have been suggested for treating the semantics of generics.

In all of the approaches to the semantics of generics we discuss, there is a fundamental reliance on a notion of *kind*. Carlson (1977b), in his seminal work on generics, noted several phenomena which provide motivation for a kind-as-individual approach. First, some some predicates appear to select generic complements, for instance *be extinct*, *be common*, and *be widespread*. Second, pronouns can have generic antecedents, as the following illustrates.

(74) Students are busy. They read and write.

Here the pronoun *they* is connected to a generic antecedent, something which is not possible with a quantified antecedent such as *every student*.

Carlson (1977b) introduced the notion of *kind* as a primitive, along with a relation between individuals and their kinds. He then interpreted generics as applying properties to these kinds. Thus (71)a might be represented as something like $\text{reptile}_g(\text{snake}_k)$, where snake_k is the kind of snakes and reptile_g is the property that applies to kinds that are reptiles. Such an analysis is rather unsatisfying without further elaboration of the connection between the two kinds of predication, generic and ordinary. Carlson relates these by a *realization relation* $\text{realize}(k)(x)$ which holds if k refers to a kind of which x is an instance. Then the non-generic interpretation of *snake* can be given by $\text{realize}(\text{snake}_k)$, which is a property that holds of all individual snakes. But rather than locating the ambiguity in the noun phrase, Carlson notes examples such as the following and claims they are evidence for an approach where the distinction is made in the verb phrase.

(75) Snow is white and falling in my yard.

Thus Carlson locates the realization relation in the verb phrase because there is only one noun phrase in (75) and we need to read one verb phrase generically and the other referentially. But our type-logical approach allows us to analyze coordinations such as (75) with a realization operator that would apply directly to the noun phrase by means of a technique we introduce in Figure 8.7 in Section 8.4. Carlson also provides a

means of translating a predicate of individuals to a predicate of kinds. Further complicating his ontology, he introduces a third class of objects, which he calls *stages*. The idea is that an individual has various stages, such as Sandy during her 112th week of life, and the tree outside Sandy's window for the next few minutes. Individuals are also related to their stages by a realization relation. The need for such operators to massage meanings into the appropriate forms present substantial obstacles to a compositional semantic analysis.

Chierchia (1982) elaborates, revises, and extends Carlson's theory by collapsing some of the type distinctions and showing how the theory can be applied to singular cases, too. His analysis is based on a type theory in which polymorphic interpretation is the norm. An elaboration of this type theory would take us far afield, but the main idea is that some of the coercions needed by Carlson's theory are either assimilated to single coercions or dispensed with entirely.

Farkas and Sugioka (1983) provide an unselective quantifier-based approach to generics, which can handle sentences with multiple generics in a natural way, such as the following.

(76) Dogs hate cats.

Generics also interact with other quantifiers, as can be seen in examples such as the following.

(77) a. Lions have a tail.

b. Students entering college have to take a test.

These examples show that generics enter into standard scope alternations, a fact not taken into account by Farkas and Sugioka's theory.

Farkas and Sugioka also provide an analysis of so-called *restrictive if/when clauses*, the canonical example of which is (78)a.

(78) a. Bears are intelligent {when / if} they have blue eyes.

b. Canaries are popular when they are rare.

The principal idea is that there is a genericity to the conditional statement. Carlson, in contrast, analyzed these cases by assuming that the restrictive clause provided a restriction on the kind, which raises substantial problems for a compositional syntax/semantics interface.

There are two fundamental problems with all of these approaches. First, they obscure the syntax/semantics relation to such an extent that it is very difficult to see how to extend the analyses to interact properly with other phenomena such as tense, quantification, anaphora, focus,

plurality, embedded contexts such as belief reports, modification, or even multiple generics. Just to take an example, consider the role of focus in the following two examples.

- (79) a. It is beavers that build dams.
 b. It is dams that are built by beavers.

Although both of these cases involve two generically interpreted bare plurals and the same relation between them, their interpretations seem quite distinct in the cleft constructions in (79). For (79)a, we are stating of dams that they are generically built by beavers; in (79)b, we alternatively state that what beavers generically build is dams. Stressing the subject or object evokes readings synonymous with those in (79) for the simple sentence *beavers build dams*.

The second problem with the analyses we have discussed is that they do not even address the deep question of what it means for a kind to have a property or what the relationship is between a kind and a property in a non-trivial way. For instance, none of these analyses explain the problem raised by Schubert and Pelletier about the notion of regularity in the interpretation of generics. Schubert and Pelletier discuss a situation in which every child born in Rainbow Lake, Alberta coincidentally happens to be right handed, and wonder about the status of the following sentence.

- (80) Babies born in Rainbow Lake, Alberta are right handed.

It seems fairly clear that genericity is in some sense tied to regular, rule-governed phenomena, itself a notoriously tricky concept to analyze. Cohen (1994, to appear) presents a theory of generics which is based on Rooth's (1985, 1992) notion of focus and a measure of probability. For instance, the comparison class in (71)b would be the class of books, in (71)c it would be other individuals who give birth, and in (71)d and (71)e, it would be other nationalities. In all cases, the generic item is statistically more likely to engage in the predicate it occurs in than the other members of the comparison class.

7.12 Comparatives

In this section, we consider the syntactic and semantic forms of *comparative* constructions. Comparison is realized in many different syntactic categories, including manner adverbs, temporal adverbs, adjectives,

determiners, and prepositions. The most widely studied class consists of the *gradable adjectives*, which, like other gradable modifiers (see (82)), can be classified into four main constructions.

- (81) a. Terry is tall. (Positive)
 b. Terry is as tall as Chris. (Equative)
 c. Terry is taller than Chris. (Comparative)
 d. Terry is the tallest child. (Superlative)

Comparatives in other categories include the following.

- (82) a. Terry wrote more prosaically than Chris. (Manner Adverb)
 b. Terry arrived later than Chris. (Temporal Adverb)
 c. Terry is taller than Chris. (Adjective)
 d. Terry wrote more papers than Chris. (Determiner)
 e. Terry lives closer to school than Chris. (Preposition)

In all of these cases, a gradable modifier (or determiner) is involved. Other categories also allow equative and superlative gradable forms. We begin our study with a brief survey of the semantics of gradability and comparison.

7.12.1 Gradability and Measurability

Fundamentally, comparison involves an ordering of a class of objects along a fixed dimension. Most abstractly, we can assume that the objects being compared form a pre-ordering (a set with a binary relation that is transitive and reflexive; see Section A.3 for full definitions). The reason we do not enforce linearity is that it might be the case that two objects cannot be compared along some dimension. For instance, in measuring how good a paper in linguistics is, many factors come into play, and it might not be possible to say that a given paper in phonology is better, worse, or even equally as good as another paper in pragmatics. Alternatively, we might compare the size of boxes and find some are longer but narrower than others; of course, if the scale is set to be volume, such comparisons become felicitous. Similarly, we do not enforce antisymmetry because we might be able to compare two objects and find out they match on a scale. We might compare two different groups along the dimension of cardinality, such as practicing phonologists and 19th century Romance literature critics, and determine that they are identical in terms of their number of members. All we need

to determine the truth of comparatives such as those in (81) is the ability to compare elements pairwise along some dimension. In general, we cannot thus be ensured that there will be a largest (or smallest) element in such a set, nor even in a finite subset of it.

Many, but not all, comparative constructions also allow *measure* or *degree* specifiers.

- (83) a. Sandy is six feet tall.
 b. Sandy is fifty IQ points smarter than Terry.
 c. Sandy's time is the fastest by three seconds.

Here the expression *six feet* provides a specification of height in terms of the combination of the number *six* and unit *feet*. Similarly, *fifty IQ points* and *three seconds* provide a numerical specifier and an indication of the units of measure. Such specifiers can be *additive measures*, as in the previous example, or *multiplicative measures*, as in the following.

- (84) a. Sandy is three times as old as Terry.
 b. That route is half as fast as this one.

Note that additive measures occur with positive, comparative, and superlative constructions, whereas multiplicative measures occur with equatives. Further note that many dimensions of measure do not allow measurement. Even though we may be able to determine that Sandy is a better writer than Terry, it is not clear that there are any objective standards, numerical or otherwise, by which to judge such matters. Notice that many multiplicative measures that do occur, such as *twice as smart* and *half as faithful*, are often meant metaphorically or hyperbolically rather than literally.

For measurable dimensions of comparison D , we can assume that there is a function $Meas_D: \mathbf{Obj} \rightarrow \mathbf{R}$ from the objects of comparison to real numbers in some fixed scale. Such a function determines a linear pre-ordering of the objects in the domain (with a linear pre-ordering, every pair of objects is such that one is bigger than the other or they are the same size). We simply set $a \preceq b$ if and only if $Meas_D(a) \leq Meas_D(b)$. Viewed from the opposite perspective, a measure function is a homomorphism from the abstract ordering of objects to the real numbers. Other scales can be normalized to a single standard to allow comparison. For instance, meters, fathoms, hands, inches and furlongs are all interconvertible, differing only by a linear multiple. It is also crucial to note that measurability is only determined relative to a scale.

For instance, consider the following.

- (85) Great Britain is larger than Australia (in terms of {population / area}).

If we are comparing populations, this is true, but if we are comparing land mass, it is false. The dimension of comparison is often supplied contextually, but can be made explicit with phrases such as *in terms of population*.

Even numerically scaled domains can have complex behaviors. For instance, the Richter scale for measuring the energy of earthquakes is logarithmic (base 60); an earthquake measuring 7 on the Richter scale generates 60 times as much energy as one measuring 6 on the Richter scale. Thus an expression such as *twice as big an earthquake* is highly ambiguous without a further specification such as *as measured on the Richter scale* or *in terms of the energy generated*. Other complexities arise from negative measures. For instance, the net worth of a person can be either positive or negative, depending on whether they have established net equity or net debt. It is rather unclear how we would use multiplicative measures to compare someone in debt to someone with net equity.

7.12.2 The Grammar of Comparatives

Over the years, several approaches to the syntax and semantics of comparatives have developed, with no clear consensus emerging. This is due to the fact that certain comparative constructions involve predicative constructions with complicated word order and distribution, along with subtle interactions with unbounded dependencies respecting island constraints, ellipsis, quantificational force, negative polarity items, vagueness, and cross-categorial similarities, in addition to a great deal of contextually supplied information which confounds the assignment of semantic types to syntactic categories. A fairly comprehensive survey of the syntactic issues and classification of syntactic constructions in transformational terms can be found in the seminal (Bresnan 1973). Typological surveys of comparative constructions can be found in (Ultan 1972; Stassen 1985). A detailed study of comparatives in Dutch in a categorial setting, concentrating on gapping and coordination, can be found in (P. Hendriks 1995).

Coupled with the rather complex nature of comparison and measure across multiple dimensions and subdomains, comparatives present a

formidable puzzle for compositional semantic treatments. Surveys of the key semantic issues can be found in (von Stechow 1984; Klein 1990). In this section, we will concentrate on the core constructions involved in the adjectival comparative constructions. Constructions in other syntactic domains are analogous, and we return to them in several exercises in later chapters after introducing the relevant non-comparative analyses (Exercise (8-21), Exercise (9-21), and Exercise (12-23)).

We will be adopting an *extent*-based approach to comparison, following Seuren (1973). Under this theory, gradable adjectives such as *tall* will be interpreted as binary relations which hold of an individual and a degree if the individual has a height of that degree or more. Although the literature is rather silent on the matter of *polar opposites* of gradable adjectives, we will interpret *short* as a relation that holds between an individual and a degree if the individual has a height which is at most that degree. Thus for the examples we will consider, we make the following definitions.

- (86) a. $\text{tall} \stackrel{\text{def}}{=} \lambda d. \lambda x. \text{height}(x) \geq d$
 b. $\text{short} \stackrel{\text{def}}{=} \lambda d. \lambda x. \text{height}(x) \leq d$

Here we assume that **height** is a measure function mapping individuals into their heights measured in meters. Thus if $\text{tall}(d)(x)$ holds, then the individual denoted by x has a height of at least the degree denoted by d . Similarly, if $\text{short}(d)(x)$ holds, then x has at most height d . As will soon become evident, the extent-based approach has several advantages over an approach in which individuals are directly associated with unique heights; for explorations of the latter approach, see (Cresswell 1976; von Stechow 1984).

It will be important later to have other functions such as **width** that map an individual to their width, and so on. It is important to note that width and height are *commensurable measures*, both given in meters. But not all measures are commensurate in this sense. This can be seen by noting the contrast in felicity created by the following examples, syntactic details of which are given following (114).

- (87) a. Sandy is wider than Terry is tall.
 b. ? Sandy is a better runner than Terry is a jumper.
 c. # Sandy is thinner than Terry is loyal.

When using commensurate degrees, the examples are fine. Perhaps (87)b can be interpreted as meaning that Sandy is in a higher percentile

as a runner than Terry is as a jumper. Using percentile scores provides one way to make all comparisons commensurable. But when the gradable adjectives lack any kind of semantic or contextual connection, as in (87)c, comparisons are less felicitous.

With an extent-based approach, we can provide a uniform semantic assignment to intensifiers that can be used with both the upward and downward gradable adjectives. First note that in the interpretation of expressions such as *is tall* or *is very tall*, there is both vagueness and context dependency. Vagueness, because it is not always clear what the boundary between tall and not-tall is, and context dependency because height is usually measured against some contextually given standard of comparison. We will represent the comparison class as a variable P denoting a set of individuals. A predicate such as *very tall* will be represented as applied to an individual variable y as $\text{very}(\text{tall})(P)(y)$ which holds if and only if $y \in P$ and there is a degree d such that $\text{tall}(d)(y)$ and very few of the other elements $x \in P$ are such that $\text{tall}(d)(x)$. Of course, what constitutes “very few” in an utterance of the intensifier is left vague. In this explanation of the intended meaning of *very*, note that there is an implicit quantification over degrees. We can codify this intuitive picture by defining the intensifier in terms of the vague determiner *few* as follows.

$$(88) \text{ very} \stackrel{\text{def}}{=} \lambda P. \lambda G. \lambda y. P(y) \wedge \text{some}(\lambda d. G(d)(y) \wedge \text{few}^2(P)(\lambda x. G(d)(x)))$$

In this term, P is the comparison class, G is a gradable adjective, and y is an individual of which the degree of G is predicated relative to P . Note that the conjunct $P(y)$ ensures that y is a member of the comparison class P . The quantificational conjunct can be glossed as stating that there is some degree d to which y has G and for which few other members of P have.

Now consider the intensified downward gradable adjective expression *very short*. The term $\text{very}(\text{short})(P)(y)$ will be true if and only if y is a member of the comparison class P that is at most some height d such that very few of the members of P are of at most height d .

For the bare predicate case, *tall*, we can introduce a predicate **pos** such that $\text{pos}(P)(G)(y)$ is true of a gradable adjective G , comparison class P and individual y , if y is in P and there is a degree d such that $G(d)(y)$ and at most half of the $x \in P$ are such that $G(d)(x)$. The positive case **pos** could be defined in the same way as the intensifier **very**.

$$(89) \text{ pos} \stackrel{\text{def}}{=} \lambda P. \lambda G. \lambda y. P(y) \wedge \text{some}(\lambda d. G(d)(y) \wedge \text{most}^2(P)(\lambda x. G(d)(x)))$$

We will need additional syntactic and semantic types in order to deal with comparatives. To simplify our exposition, we will only consider the dimension of length, as expressed by predicates such as *tall*, *short*, *long*, and *wide*. We will also assume that meters are the units of measurement; other linear length scales can be mapped linearly to meters, as exemplified below in our treatment of unit-denoting terms such as *inches* and *fathoms*. Thus we will not need to be concerned about the kind of vagueness with respect to dimension expressed in (85), nor in abstract comparison classes expressed solely as partial orders. The categories we assume and their associated types are as follows.

(90)	<i>Category</i>	<i>Type</i>	<i>Description</i>
	<i>num</i>	Real	real number
	<i>deg</i>	Real	degree

We have simplified matters by assuming assuming that we are dealing with a single kind of degree measurable with real numbers. For a more general treatment, degrees need to include not only a numerical value, but also an indication of the *dimension* of comparison, such as weight, height, or land speed. For non-measurable degrees, we could follow Cresswell (1976) in representing degrees as equivalence classes of objects ordered by the standard induced ordering (see Section A.3). We also assume that the standard arithmetic operations of multiplication and addition interpret the constants **times** and **plus**, which we will abbreviate with the standard infix operators \cdot and $+$.

We begin with an analysis of the following three predicative instances of gradable adjectives, in both upward and downward polar varieties.

- (91) a. Sandy is 160 centimeters {tall / # short}.
 b. Sandy is {tall / short}.
 c. Sandy is {very / extremely} {tall / short}.
 d. * Sandy is {160 cm very / very 160 cm} tall.

Unlike the positive gradable adjective, *tall*, the negative *short* can be used bare or with a degree modifier, but not with an explicit degree (except in an ellided form in which it implies failure to reach some measure, as in *Sandy is 5 centimeters short of {two meters / her goal}*). This distribution could be coded straightforwardly with syntactic features. It is not at all clear why explicit degrees are infelicitous with the negative

gradable adjectives in the context of an extent-based theory. The property *160 centimeters short* would apply to an individual which is at most 160 centimeters tall. We will see further reasons below to distinguish positive from negative gradable adjectives in the context of equative and comparative constructions. Finally note that degree specifiers and intensifiers cannot co-occur, as will be reflected in our type assignments.

The following lexical entries generate the correct meanings for the positive instances of gradable adjectives, such as those in (91).

- (92) a. *tall* \Rightarrow *tall*: *deg*\ *pr*
 b. *short* \Rightarrow *short*: *deg*\ *pr*
 c. *160* \Rightarrow *160*: *num*
 d. *centimeters* \Rightarrow $\lambda x.(0.01 \cdot x)$: *num*\ *deg*
 e. *tall* \Rightarrow *pos*(*X*)(*tall*): *pr*
 f. *very* \Rightarrow *very*(*X*): *pr*/(*deg*\ *pr*)

We employ two lexical entries for *tall*. In (92)a, *tall* requires a degree specifier to produce a proposition indicating that its subject is at least as tall as the degree specifier. This usage is illustrated in Figure 7.27. Note that the unit expression *centimeters* is assigned a lexical entry in (92)d in which it takes a numerical complement, such as the one in (92)c, in order to produce a degree. We have ignored number agreement in the syntax; the singular degree expression is appropriate for *one meter* and *one half meter*, whereas the plural degree expression is found in *two meters* and *many meters*. The entry (92)e does not take a degree complement, but merely specifies that the subject is above average relative to a contextually supplied class *X*, which we have indicated rather sloppily as a variable. An example derivation with this lexical assignment can be found in Figure 7.28. In some cases, the contextually given comparison class can be indicated directly with expressions such as *for a boy*, although the manner in which such a complement expresses a set is non-trivial; perhaps it involves a reference to kinds (see Section 7.11). In Figure 7.29, we provide a derivation involving the intensifier lexical entry in (92)f. Note that the derived meaning for an intensified gradable adjective also involves an implicit comparison class.

With the categories we have assigned to intensifiers such as *very*, they will not iterate, thus blocking well-formed expressions such as *very very tall* and *really really short*. We claim that such occurrences should be treated by a lexical reduplication of some sort, for two reasons.

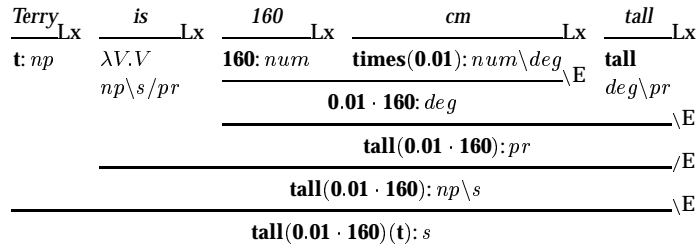


Figure 7.27
Analysis of *Terry is 160 cm tall*

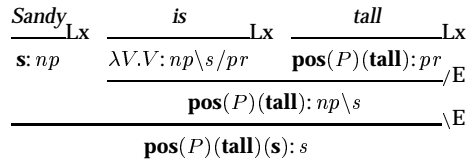


Figure 7.28
Analysis of *Sandy is tall*

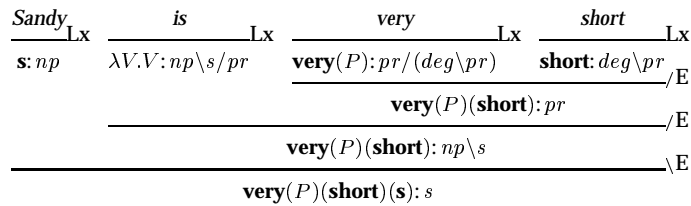


Figure 7.29
Analysis of *Sandy is very short*

First, many intensifiers do not iterate, as in **extremely extremely tall* or **quite quite quite tall*. Second, the intensifiers that do iterate do not mix, as witnessed by the infelicity of **very really very really tall* (the intensifier form of *really* should not be confused with the adverbial usage, as in *really very tall*, which is perfectly acceptable, in contrast to **very really tall*).

One issue worth considering at this point is the notion of *scalar implicature*, which is a kind of Gricean implicature having to do with scalars. Grice's maxims explain why we prefer *Sandy is six feet tall* to *Sandy is five feet tall*, if Sandy is in fact six feet tall. With an extent-based approach to gradable adjectives, the property of being six feet tall entails the property of being five feet tall, but not vice-versa. In other words,

although adjectives with degree specifiers provide only a lower bound, we are only being cooperative if we provide as tight a lower bound as possible. Of course, in some circumstances, the exact height may be irrelevant as long as it is above a certain minimum. For instance, we might say that *Sandy is four feet tall* if Sandy is in fact four feet, five inches tall, if four feet is the minimum height required to ride a roller coaster at an amusement park. This completes our brief survey of the positive occurrences of gradable adjectives.

We turn now to the equative uses of gradable adjectives. We will be analyzing the following cases.

- (93) a. Sandy is as {tall / short} as Terry.
 b. Sandy is {twice / half / two times} as {tall / ? short} as Terry.

We will treat the complement *as Terry* as a noun phrase object case marked by *as*. In this case, we need to deal with multipliers rather than degrees. The following logical constant for multiplicative height relations will be useful.

$$(94) \text{mult} \stackrel{\text{def}}{=} \lambda n. \lambda G. \lambda x. \lambda y. \text{every}^2(\lambda d. G(d)(x))(\lambda d. G(n \cdot d)(y))$$

Thus $\text{mult}(n)(G)(x)(y)$ will hold if for every degree to which x is G , y has G to degree at least n times that. Note that in multiplication, the forms of numerals vary; thus we find expressions like *twice* and *two times* rather than either bare numerals like *two* or degrees like *two inches*. To this end, we introduce the following category for multipliers.

(95)	<i>Category</i>	<i>Type</i>	<i>Description</i>
	<i>mlt</i>	Real	Multiplier

The examples in (93) can be derived from the following lexical entries.

- (96) a. *as* $\Rightarrow \lambda x. x: np_{as}/np$
 b. *as* $\Rightarrow \text{mult}(1): pr/np_{as}/(deg \setminus pr)$
 c. *as* $\Rightarrow \text{mult}: mlt \setminus (pr/np_{as}/(deg \setminus pr))$
 d. *twice* $\Rightarrow 2: mlt$
 e. *times* $\Rightarrow \lambda n. n: num \setminus mlt$

Derivations with and without an explicit numerical multiplier are given in Figure 7.30 and Figure 7.31. Note that the only difference is that the bare case has the constant **1** as an implicit multiplier.

Just as in the positive cases, the notion of scalar implicature explains why we want to provide as tight a bound as possible on someone's

$$\begin{array}{c}
\frac{Kim_{Lx} \quad is_{Lx} \quad as_{Lx} \quad tall_{Lx} \quad as\ Sandy_D}{k: np \quad \lambda V.V: np \backslash s / pr \quad \mathbf{mult}(1): pr / np_{as} / (deg \backslash pr) \quad \mathbf{tall}: deg \backslash pr \quad s: np_{as}} \\
\hline
\mathbf{mult}(1)(\mathbf{tall}): pr / np_{as} \\
\hline
\mathbf{mult}(1)(\mathbf{tall})(s): pr \\
\hline
\mathbf{mult}(1)(\mathbf{tall})(s): np \backslash s \\
\hline
\mathbf{mult}(1)(\mathbf{tall})(s)(k): s
\end{array}$$

Figure 7.30
Analysis of *Kim is as tall as Sandy*

$$\begin{array}{c}
\frac{Kim_{Lx} \quad is_{Lx} \quad two_{Lx} \quad times_{Lx} \quad as_{Lx} \quad tall_{Lx} \quad as\ Sandy_D}{k: np \quad \lambda V.V: np \backslash s / pr \quad \mathbf{2} \quad \lambda n.n \quad \mathbf{mult} \quad \mathbf{tall} \quad s: np_{as}} \\
\frac{num \quad num \backslash mlt}{\mathbf{2}: mlt} \\
\hline
\mathbf{mult}(2): pr / np_{as} / (deg \backslash pr) \\
\hline
\mathbf{mult}(2)(\mathbf{tall}): pr / np_{as} \\
\hline
\mathbf{mult}(2)(\mathbf{tall})(s): pr \\
\hline
\mathbf{mult}(2)(\mathbf{tall})(s): np \backslash s \\
\hline
\mathbf{mult}(2)(\mathbf{tall})(s)(k): s
\end{array}$$

Figure 7.31
Analysis of *Kim is two times as tall as Sandy*

height. If Kim is 2.3 times as tall as Sandy, it is truthful to say that Kim is twice as tall as Sandy, but it is possible to be more informative if the situation warrants it. Such considerations are crucial in interpreting expressions involving fractional multipliers in equatives, such as the following.

(97) Kim is half as tall as Sandy.

Our grammar derives an interpretation which will be true if Kim is at least half as tall as Sandy. But again, if Kim were in fact exactly 75% as tall as Sandy, it would simply be more informative to assert that directly. Note that our analysis also accounts for negative occurrences, such as the following.

(98) Kim is not half as tall as Sandy.

Such a sentence can be uttered truthfully only if there is some degree

d forming a lower bound on Sandy's height such that Kim is not half of d tall. Some uses of the negative case may be even more subtle in that we may be negating pragmatic conditions rather than semantic ones; that is, we might use the previous sentence if Kim and Sandy are the same heights, thus negating the scalar implicature. Matters are further complicated because sentences such as (98) can alternatively be phrased positively by *Sandy is less than half as tall as Sandy*. Negation and its interaction with discourse is a thorny issue, the best approach to which has been provided by Horn (1985, 1989).

Next, consider the equative uses of the polar opposites of gradable adjectives, with and without multiplicative specifiers.

- (99) a. Sandy is as short as Terry.
 b. Sandy is {half / twice} as short as Terry.

In the non-specific case, (99)a, we derive a meaning whereby every upper bound on Terry's height is also an upper bound on Sandy's height. Thus if Terry is at most five feet tall, then Sandy is also at most five feet tall. Although this interpretation seems perfectly natural, it is not at all clear, even at an intuitive level, how to interpret the cases in (99)b with multiplicative specifiers. But it is clear that our grammar assigns non-sensical meanings. In the case of *twice*, we derive an interpretation whereby every upper bound d on Terry's height is such that $2d$ is an upper bound on Sandy's height, thus allowing Sandy to be taller than Terry. The situation for *half* is no better. Perhaps such uses, when felicitous, are simply being used metaphorically or in terms of coercion to a positive percentile scale.

We now turn to the comparative uses of gradable adjectives. Consider the following examples.

- (100) a. Sandy is taller than Terry.
 b. Sandy is six centimeters taller than Terry.

Note that the operation applied to the explicit degree given in (100)b is addition. But for the bare case of (100)a, we must have some number greater than 0, in order to rule out the case where Sandy and Terry are exactly the same height. This is yet another way in which the equative and comparative diverge semantically. The following logical constants will be used in the lexical specification of the positive comparative forms.

- (101) a. $\text{add} \stackrel{\text{def}}{=} \lambda e. \lambda G. \lambda x. \lambda y. \text{some}(\lambda d. G(d + e)(y) \wedge \neg G(d)(x))$

$$\begin{array}{c}
\frac{\frac{\frac{Kim_{Lx}}{k: np} \quad \frac{is_{Lx}}{\lambda V.V} \quad \frac{two\ cm_D}{0.01 \cdot 2}}{np \setminus s / pr} \quad \frac{tall_{Lx}}{deg}}{deg \setminus pr} \quad \frac{-er_{Lx}}{\lambda G. \lambda d. \mathbf{add}(d)(G)}}{\lambda d. \mathbf{add}(d)(\mathbf{tall}): deg \setminus (pr / np_{th})} \setminus E \\
\frac{\mathbf{add}(0.01 \cdot 2)(\mathbf{tall}): pr / np_{th}}{\mathbf{add}(0.01 \cdot 2)(\mathbf{tall})(s): pr} / E \\
\frac{\mathbf{add}(0.01 \cdot 2)(\mathbf{tall})(s): np \setminus s}}{\mathbf{add}(0.01 \cdot 2)(\mathbf{tall})(s)(k): s} \setminus E
\end{array}$$
Figure 7.32Analysis of *Kim is two cm taller than Sandy*

$$b. \mathbf{addsome} \stackrel{\text{def}}{=} \lambda G. \lambda x. \lambda y. \mathbf{some}(\lambda e. \mathbf{add}(e)(G)(x)(y))$$

Note that in these terms e and d are degrees, G a gradable adjective, and x and y individuals in this term. Thus $\mathbf{add}(e)(G)(x)(y)$ will hold if there is some degree d to which x has G to the degree $d + e$, but y does not have G even to the extent d . Thus x will have G to some degree at least e greater than y does. The term **addsome** simply existentially binds the degree argument of **add**.

In expressing the comparative, we find an alternation between the suffix *-er* and the syntactic modifier *more* (which also has a polar opposite form, namely *less*, which we consider in Exercise (7-25)). The facts concerning the distribution of these two forms are subtle and depend at least on matters of morphological and metrical structure (stress and length, in particular). We treat the complement to the comparative as a *than*-marked noun phrase.

$$(102) a. \mathbf{more} \Rightarrow \lambda G. \lambda d. \mathbf{add}(d)(G): deg \setminus (pr / np_{th}) / (deg \setminus pr)$$

$$b. \mathbf{-er} \Rightarrow \lambda G. \lambda d. \mathbf{add}(d)(G): (deg \setminus pr) \setminus deg \setminus (pr / np_{th})$$

$$c. \mathbf{more} \Rightarrow \mathbf{addsome}: pr / np_{th} / (deg \setminus pr)$$

$$d. \mathbf{-er} \Rightarrow \mathbf{addsome}: (deg \setminus pr) \setminus (pr / np_{th})$$

These lead to derivations with explicit degrees, as in Figure 7.32, and with existentially bound degrees, as in Figure 7.33. Note that just like the equative case, the meaning assigned to the comparative morpheme and modifier extends naturally to the downward gradable adjectives.

For cases of polar negative gradable adjectives, such as *short*, we can provide roughly the same kind of analysis. The one difference is that we use subtraction rather than addition in the underlying semantics

$$\begin{array}{c}
 \frac{\text{Kim}_{LX} \quad \frac{\text{is}_{LX} \quad \frac{\text{tall}_{LX} \quad \frac{-er_{LX} \quad \text{than Sandy}_D}{\text{s: } np_{th}}}{\text{deg}\backslash pr} \backslash E}{\text{deg}\backslash pr} \backslash E}{\text{deg}\backslash pr} \backslash E}{\text{deg}\backslash pr} \backslash E}{\text{deg}\backslash pr} \backslash E}{\text{deg}\backslash pr} \backslash E} \\
 \text{k: } np \quad \lambda V.V \quad \text{tall} \quad \text{addsome} \quad \text{s: } np_{th} \\
 np\backslash s/pr \quad \text{deg}\backslash pr \quad (\text{deg}\backslash pr)\backslash (pr/np_{th}) \backslash E \\
 \text{addsome(tall): } pr/np_{th} \\
 \text{addsome(tall)(s): } pr \\
 \text{addsome(tall)(s): } np\backslash s \\
 \text{addsome(tall)(s)(k): } s
 \end{array}$$

Figure 7.33
Analysis of *Kim is tall er than Sandy*

of the comparative morpheme or modifier. Of course, this should be syntactically indicated in some way, but we will finesse that detail here. Consider the meanings that would be assigned under such a move.

$$(103) \text{ Kim is 3 cm shorter than Sandy} \Rightarrow \\ \text{some}(\lambda d.\text{short}(d - 3 \text{ cm})(k) \wedge \neg \text{short}(d)(s))$$

Similar readings would arise in the bare case by existentially quantifying over the degree of difference.

One further point to note about our analysis in terms of extents has to do with the boundary conditions. We defined extents in terms of the less-than-or-equal comparison. Unfortunately, in the case of comparatives, this rules out an “exact” reading of the degree of difference e . For instance, if we have $\text{tall}(d + e)(s) \wedge \neg \text{tall}(d)(k)$, then Sandy cannot be exactly e taller than Kim, because of the negation. That is, Sandy is at least $d + e$ tall, but Kim is not even d tall. Although the difference is arbitrarily small, it would perhaps be better to place the burden of extent-based reasoning on the comparative and equative predicates rather than on the gradable adjective. We explore such an alternative in Exercise (7-31).

We now turn to an analysis of superlatives. The superlative construction is marked with either an *-est* suffix, or with the modifier *most*.

- (104) a. Sandy is tallest.
b. Sandy is most aggressive.

These examples are rather stilted when used as predicates, but appear more natural when used as postnominal modifiers (see (107) below). The notion of superlative degree is that the individual in question is at the top of the scale, relative to some contextually supplied comparison

$$\begin{array}{c}
 \frac{\text{Sandy}_{Lx} \quad \frac{\text{is}_{Lx} \quad \frac{\text{short}_{Lx} \quad \frac{\text{est}_{Lx}}{\text{top}(R):(deg\backslash pr)\backslash pr} \backslash E}}{\text{short}: deg\backslash pr} \backslash E}}{\text{top}(R)(\text{short}): pr} \backslash E}}{\text{top}(R)(\text{short}): np\backslash s} \backslash E}}{\text{top}(R)(\text{short})(s): s} \backslash E}
 \end{array}$$

Figure 7.34
Analysis of *Sandy is shortest*

class. The following constant can be defined to serve this function.

$$(105) \text{ top} \stackrel{\text{def}}{=} \lambda P. \lambda G. \lambda x. P(x) \wedge \text{some}(\lambda d. G(d)(x) \wedge \text{every}(\lambda y. P(y) \wedge y \neq x) (\lambda y. \neg G(d)(y)))$$

With this definition, $\text{top}(P)(G)(x)$ will be true if x is an individual in P such that there is some degree to which x is tall but no other member of P is that tall (we return to consider the superlative in the plural form, as in *tallest three boys*, in Exercise (8-24)). Note that this definition transfers appropriately to the downward gradable adjectives like *short*. This is because having an upper bound on height which no one else has makes one the shortest person.

We can now provide the lexical entries for the superlative suffix and modifier.

$$(106) \text{ a. } -est \Rightarrow \text{top}(P): (deg\backslash pr)\backslash pr \\
 \text{ b. } most \Rightarrow \text{top}(P): pr/(deg\backslash pr)$$

A sample derivation can be found in Figure 7.34.

All of the forms of gradable adjective can also be used as nominal modifiers, as the following examples illustrate.

- (107) a. (160 centimeter) tall student
 b. * student tall
 c. student 160 centimeters tall
 d. student (twice) as tall as Sandy
 e. (* Six cm) taller student than Sandy
 f. student (six cm) taller than Sandy
 g. tallest student

Note that there are subtle variations in word order. The positive construction can be used prenominal, with or without an explicit degree,

as seen in (107)a. The example in (107)b shows that a simple positive gradable predicate cannot appear postnominally; but (107)c appears better, although note the pluralization of the degree. The equative only appears postnominally, with or without a multiplier, as seen in (107)d. The comparative can appear prenominal without an explicit degree, and postnominally with or without the degree specifier. The semantics of the modifier forms is derived from the predicate form by intersection. We leave the specifics to Exercise (8-21). The only semantic facts worth noting concern the contribution of the modified noun. By simply constructing intersective modifiers out of the predicate forms, we assume that the comparison class is always given contextually and not forced to be identified with the noun being modified. Following Pollard and Sag (1994), our claim that the other instances are just the natural intersective modifiers leads to the conclusion that the comparison class for the positive constructions is not required to be the interpretation of the noun being modified. They consider an example of a linguistics department recruiting intramural basketball players, during which time someone might say they were looking for a *good linguist* with the meaning that the linguist was a good basketball player and not necessarily a good linguist. Of course, it would be a simple matter semantically to restrict the lexical entry so that the modified noun supplied the comparison class in the positive and superlative cases. Further note that there is an implication in (107)e that Sandy must be a student (in general, the comparative object must satisfy the property given by the modified noun); there is no such implication made by (107)f.

It is interesting to note that the extent-based constants we have used to derive meanings for the equative *as tall* and the comparative *taller* create a negative polarity environment for the object noun phrase. The downward monotonic environment is created by the negation of an extent-based degree predicate, as seen in the derivations of the equative Figure 7.30 and the comparative Figure 7.33, which expand as follows.

- (108) a. *Kim is as tall as Sandy* \Rightarrow
 $\text{some}(\lambda d. \text{tall}(d)(k) \wedge \neg \text{tall}(d)(1 \cdot s)); s$
- b. *Kim is taller than Sandy* \Rightarrow
 $\text{some}(\lambda e. \text{some}(\lambda d. \text{tall}(d + e)(k) \wedge \neg \text{tall}(d)(s))); s$

For the superlative modifier, it seems that examples like *the best student who anyone has ever met* indicate that the whole of the noun and its relative clause modifier should constitute a negative polarity context.

$Sandy$	is	$tall$	$-er$	$than Kim is$
$s: np$	$\lambda V.V$ $np \setminus s / pr$	$tall$ $deg \setminus pr$	$\lambda G. \lambda U. \lambda y. U(\lambda x. \mathbf{addsome}(G)(y)(x))$ $(deg \setminus pr) \setminus (pr / (s \uparrow pr))$	$\lambda R. R(\mathbf{k})$ $s \uparrow pr$
			$\lambda U. \lambda y. U(\lambda x. \mathbf{addsome}(tall)(x)(y))$ $pr / (s \uparrow pr)$	
			$\mathbf{addsome}(tall)(\mathbf{k}): pr$	
			$\mathbf{addsome}(tall)(\mathbf{k}): np \setminus s$	
			$\mathbf{addsome}(tall)(\mathbf{k})(s): s$	

Figure 7.36Analysis of *Sandy is taller than Kim is*

Such derivations follow the other argument raising cases we have considered. Note that the same kind of raising could be applied to the objects of the entries without explicit degrees and multipliers. But what is interesting is that these derived meanings are just what we need to provide lexical entries for the comparative deletion cases.

- (111) a. $as \Rightarrow \lambda G. \lambda n. \lambda Q. \lambda x. Q(\lambda y. \mathbf{mult}(n)(x)(y))$
 $mlt \setminus (pr / (s \uparrow pr)) / (deg \setminus pr)$
- b. $more \Rightarrow \lambda G. \lambda d. \lambda Q. \lambda x. Q(\lambda y. \mathbf{add}(d)(G)(y)(x))$
 $deg \setminus (pr / (s \uparrow pr)) / (deg \setminus pr)$

The beauty of Larson's analysis of comparative deletion is that it interacts appropriately with quantifier scope. Consider the contrast in scope possibilities between the following two examples.

- (112) a. Someone is taller than everyone.
 b. Someone is taller than everyone is.

In the first example, (112)a, the object quantifier *everyone* can take wide or narrow scope. With the object given widest scope, (112)a means that for every person, there is someone who is taller than them; with the object scoped narrowly, it means that there is one person who is taller than everyone. These two scopes can be generated in the ordinary way using our quantifier elimination schemes. The example (112)b, on the other hand, does not have a reading in which the quantifier embedded in the $s \uparrow pr$ complement receives widest scope. This is to be expected because the sentential complement is an island to extraction, as noted by Chomsky (1977). The correct scoping is derived with the following analysis of the complement, which proceeds along similar lines to the

$$\begin{array}{c}
 \frac{\text{Sandy}_{Lx} \quad \text{is}_{Lx} \quad [d:deg]^0 \quad \text{tall}_{Lx}}{\text{s: np} \quad \lambda V.V \quad \text{np} \backslash s / pr \quad \text{tall} \quad deg \backslash pr \backslash E} \\
 \frac{\text{tall}(d): pr}{\text{tall}(d): np \backslash s} \backslash E \\
 \frac{\text{tall}(d)(s): s}{\lambda d. \text{tall}(d)(s): s \uparrow deg} \uparrow I^0
 \end{array}$$

Figure 7.37
Subdeletion Analysis of *Sandy is tall*

derivation of *everyone likes* in Figure 7.15.

$$(113) \text{ everyone is} \Rightarrow \lambda P. \text{every}(P): s \uparrow pr$$

The wider scoping of the embedded quantifier is then blocked by whatever mechanism blocks other island violations (see Morrill 1992b; 1994a). The equative entries with clausal complements with extracted elements can be treated in the same way.

Unlike comparative deletion, *comparative subdeletion* only involves the omission of the degree modifier rather than the entire predicate.

- (114) a. Sandy is (twice) as tall as [Terry is __ wide].
b. Sandy is (6 centimeters) wider than [Terry is __ tall].

The complements will thus be given the category $s \uparrow deg$ of a sentence with a degree expression extracted. An example of a derivation of such a category can be found in Figure 7.37. Subdeletion requires another lexical entry for both the equative and comparative morphemes and modifiers. These can be formulated as follows.

$$\begin{array}{l}
 (115) \text{ a. } as \Rightarrow \text{esd}: mlt \backslash (pr / (s \uparrow deg)) / (deg \backslash pr) \\
 \text{ b. } \text{esd} \stackrel{\text{def}}{=} \lambda G. \lambda n. \lambda V. \lambda x. \text{some}(\lambda d. G(d)(x) \wedge \neg V(n \cdot d))
 \end{array}$$

$$\begin{array}{l}
 (116) \text{ a. } more \Rightarrow \text{csd}: deg \backslash (pr / (s \uparrow deg)) / (deg \backslash pr) \\
 \text{ b. } -er \Rightarrow \text{csd}: (deg \backslash pr) \backslash (deg \backslash (pr / (s \uparrow deg))) \\
 \text{ c. } \text{csd} \stackrel{\text{def}}{=} \lambda G. \lambda e. \lambda V. \lambda x. \text{some}(\lambda d. G(d)(x) \wedge \neg V(e + d))
 \end{array}$$

The entries without explicit multiplier or degree complements can be defined as usual. In the case of equative *as* we insert the constant **1**, and in the case of comparative *more*, we existentially quantify over the degree of difference.

$\frac{\text{Sandy}}{s: np}$	$\frac{\text{is}}{\lambda V.V}$	$\frac{3\text{ cm}}{np \setminus s/pr}$	$\frac{\text{wide}}{deg}$	$\frac{\text{-er}}{deg \setminus pr}$	$\frac{\text{than Terry is tall}}{\lambda d.tall(d)(t)}$
Lx	Lx	D	Lx	Lx	D
			$\frac{\text{csd}}{(deg \setminus pr) \setminus (pr / (s \uparrow deg))}$		
			$\text{csd(wide): } deg \setminus (pr / (s \uparrow deg))$		
			$\text{csd(wide)(0.03): } pr / (s \uparrow deg)$		
			$\text{csd(wide)(0.03)(}\lambda d.tall(d)(t)\text{): } pr$		
			$\text{csd(wide)(0.03)(}\lambda d.tall(d)(t)\text{): } np \setminus s$		
			$\text{csd(wide)(0.03)(}\lambda d.tall(d)(t)\text{)(s): } s$		

Figure 7.38
Analysis of *Sandy is 3 cm wide-er than Terry is tall*

- (117) a. $as \Rightarrow \text{besd: } pr / (s \uparrow deg) / (deg \setminus pr)$
 b. $\text{besd} \stackrel{\text{def}}{=} \lambda G. \lambda V. \lambda x. \text{some}(\lambda d. G(d)(x) \wedge \neg V(\mathbf{1} \cdot d))$
- (118) a. $more \Rightarrow \text{bcsd: } pr / (s \uparrow deg) / (deg \setminus pr)$
 b. $-er \Rightarrow \text{bcsd: } (deg \setminus pr) \setminus (pr / (s \uparrow deg))$
 c. $\text{bcsd} \stackrel{\text{def}}{=} \lambda G. \lambda V. \lambda x. \text{some}(\lambda e. \text{some}(\lambda d. G(d)(x) \wedge \neg V(e + d)))$

A sample derivation involving a case of comparative subdeletion can be found in Figure 7.38. Note that the same facts concerning scope possibilities arise for the subdeletion case as arise for deletion; the complement lacking a degree is an island, but quantifiers within it can be assigned narrow scope by reducing them within the clausal construction.

7.13 Expletives and the Unit Type

In English, the words *it* and *there* in their non-pronominal incarnations do not provide a semantic contribution to meaning. First consider some uses of non-pronominal *it*.

- (119) a. It is raining.
 b. It is Francis who Brett conspired with.
 c. It bothered Pat that Francis conspired with Brett.

The first example, (119)a, involves a *weather predicate*. The second sentence, (119)b, is an instance of *clefting*. The last example, (119)c, is an

instance of sentential subject *extraposition*. Next, consider the use of *there*.

- (120) a. There is a unicorn.
 b. There was a lion in the garden.
 c. There were thirty seven papers submitted.

Each of these examples involves an existential claim. These noun phrase placeholders are known as *expletives*, *pleonastics*, or *dummies*. Their semantic analysis is puzzling from the perspective of compositional, type-logical approaches (Klein and Sag 1985; Morrill and Carpenter 1990).

The solution adopted in GPSG (Gazdar, Klein, Pullum, and Sag 1985) is to treat expletives as ordinary noun phrases whose denotation is required to be a special distinguished individual. Gazdar et al. (1985) attribute to Dowty the notion of treating the corresponding verbs as vacuously abstracting over their arguments. But expletives do not have any of the other properties of noun phrases, such as providing potential antecedents for pronouns, or participating in scoping relations. Furthermore, the assumption of a distinguished individual in the domain of individuals is ontologically unmotivated and thus rather ad hoc.

Within a type-logical approach, there is a more appealing solution, which can be formulated by using the following standard addition to the type theory.

Definition 7 (Unit Type) *The unit type, written as $\mathbf{1}$, is associated with the singleton domain $\mathbf{Dom}_1 \stackrel{\text{def}}{=} \{1\}$.*

By assuming that we have a constant $\mathbf{1}$ of type $\mathbf{1}$, we can conclude the following for every model \mathcal{M} (note that we are overloading our notation here).

$$(121) \llbracket \mathbf{1} \rrbracket_{\mathcal{M}} = 1$$

The reason $\mathbf{1}$ is called the unit type is because of its behavior as an identity (up to isomorphism) for both products and functions. More specifically, the elements of type $\mathbf{1} \times \sigma$ will all be of the form $\langle 1, a \rangle$ where a is an object of the domain of type σ . Thus the domain of $\mathbf{1} \times \sigma$ stands in a one-to-one relationship to the domain for σ . Because products are symmetric, the domain of $\sigma \times \mathbf{1}$ is also isomorphic to that of σ . For functions, note that the elements of $\mathbf{1} \rightarrow \sigma$ will be functions from the domain of type $\mathbf{1}$ to the domain of type σ . Because the domain of $\mathbf{1}$ is a singleton, each function in the domain of $\mathbf{1} \rightarrow \sigma$ only determines the image of

the object $\mathbf{1}$. There is exactly one such function for each element of the domain of type σ . Thus the domains $\mathbf{1} \rightarrow \sigma$ and σ are isomorphic in the sense of standing in a one-to-one correspondence. The function type constructor is not symmetric. Indeed, the domain $\sigma \rightarrow \mathbf{1}$ consists of mappings from the domain of σ to the singleton domain of objects of type $\mathbf{1}$. Of course, there is only one such function, making $\sigma \rightarrow \mathbf{1}$ isomorphic to $\mathbf{1}$ itself. The Curry-Howard morphism can be extended to the type $\mathbf{1}$ by associating this type with the denotation of the propositional constant **true**. That this extension is natural is evidenced by the domain equalities mentioned above. For instance, note that $\text{true} \wedge \phi$ and $\text{true} \rightarrow \phi$ are logically equivalent to ϕ . In Exercise (7-14), we explore the related *zero type* $\mathbf{0}$, whose domain is the empty set, and which corresponds to the denotation of the propositional constant **false**.

For grammatical purposes, we assume two new basic categories.

Definition 8 (Expletive Categories and Types) *The expletive categories and corresponding types are:*

- a. $ex_t, ex_i \in \mathbf{BasCat}$
- b. $Typ(ex_t) = Typ(ex_i) = \mathbf{1}$

The subscripts t and i are used to mark expletives corresponding to *there* and *it* respectively. With this definition, we ensure that an expression of type ex_t or ex_i denotes $\mathbf{1}$, because that is the only element of the appropriate domain. We assume that the expletives are of these categories.

- (122) a. $it \Rightarrow \mathbf{1}: ex_i$
 b. $there \Rightarrow \mathbf{1}: ex_t$

Because we take ex_t and ex_i to be basic categories, they may be connected with other categories. In the case of verbs taking expletive *it* as subject, the following category is appropriate.

- (123) $rained \Rightarrow \text{rain}: ex_i \setminus s$

This allows us to generate the following analysis by simple application.

- (124) $it \text{ rained} \Rightarrow \text{rain}(\mathbf{1}): s$

Because rain is of type $\mathbf{1} \rightarrow \mathbf{Bool}$, it can be identified with an element of the domain of type \mathbf{Bool} . The dummy-like nature of *it* is thus naturally accounted for; syntactically, it acts like other subjects, but semantically, it is vacuous in that it makes no contribution.

$$\begin{array}{c}
 \frac{\textit{there}}{\mathbf{1}: ex_t} \text{LX} \quad \frac{\textit{is}}{\lambda x.\lambda z.\mathbf{true}: ex_t \setminus s / np} \text{LX} \quad \frac{\textit{a unicorn}}{\mathbf{some}(\mathbf{uni}): np \uparrow s} D \\
 \hline
 \frac{\mathbf{some}(\mathbf{uni}): np \uparrow s}{y: np} \uparrow E^0 \\
 \hline
 \frac{\lambda z.\mathbf{true}: ex_t \setminus s}{\mathbf{true}: s} /E \\
 \hline
 \frac{\mathbf{true}: s}{\mathbf{some}(\mathbf{uni})(\lambda y.\mathbf{true}): s} \backslash E \\
 \hline
 \mathbf{some}(\mathbf{uni})(\lambda y.\mathbf{true}): s \quad 0
 \end{array}$$

Figure 7.40
Analysis of *there is a unicorn*

as *given* versus *new* in a discourse (Halliday 1967; Brown and Yule 1983, Chapter 5). Such matters determine, for instance, the potential for anaphoric reference.

The case of expletive *there* is similarly straightforward. Perhaps the most obvious analysis is in terms of a predicate of existence.

- (127) a. $is \Rightarrow \mathbf{some}_2: ex_t \setminus s / np$
 b. $\mathbf{some}_2 \stackrel{\text{def}}{=} \lambda x.\lambda y.\mathbf{true}$

Consider the derivation in Figure 7.40. Note that the meaning assigned in Figure 7.40 asserts the existence of a unicorn. We return to the notion of existence in Section 10.2.1, in the context of an intensional logic. Next, consider the following analysis, which only varies from that in Figure 7.40 in the meaning of the determiner.

- (128) $there\ is\ no\ unicorn \Rightarrow \mathbf{no}(\mathbf{uni})(\lambda x.\mathbf{true})$

Such a sentence can thus only be used truthfully if there are no unicorns.

Now consider the status of the following two examples, which are certainly infelicitous, if not ungrammatical, when used on their own

- (129) a. # There is Sandy.
 b. # There is every unicorn.

As answers to questions, these can both be acceptable; consider *Who can we get to deliver the letter?*, to which (129)a is a perfectly reasonable answer. This is perhaps because in answering questions, we are interested in simply contributing referents to the discourse (see Chapter 9). Note that the meaning assigned to the first example, (129)a, is simply **true**, because the object's referent is discarded after application, as shown in Figure 7.41. (Of course, there is a preferred reading of (129)a in which the subject *there* acts as a demonstrative locative pronoun.) Further, if

$$\begin{array}{c}
 \frac{\textit{there}}{\mathbf{1}: ex_t} \text{Lx} \quad \frac{\textit{is}}{\lambda x. \lambda y. \mathbf{true}: ex_t \backslash s / np} \text{Lx} \quad \frac{\textit{Sandy}}{\mathbf{s}: np} \text{Lx} \\
 \hline
 \lambda y. \mathbf{true}: ex_t \backslash s \\
 \hline
 \mathbf{true}: s
 \end{array} \text{E}$$

Figure 7.41
Analysis of *there is Sandy*

we assume interpretation is a dynamic process whereby noun phrase referents are established before they become complements to verbs, then (129)a could be understood as introducing a referent into the discourse. But the sentential meaning assigned to (129)a in Figure 7.41 is trivially true. Similarly, the meaning assigned to (129)b, $\mathbf{every}(\mathbf{uni})(\lambda z. \mathbf{true})$, is also vacuously true.

Barwise and Cooper (1981) provide a semantic explanation for the infelicity of the examples in (129). Specifically, statements are not allowed to have readings which are degenerate in being always true or always false independent of the model. This principle clearly fails in general, as can be seen with perfectly acceptable examples such as *Francis is Francis*, or *Francis is tall* or *Francis is not tall*. What remains interesting about Barwise and Cooper's approach is that they apply their general principle to provide a semantic characterization of when *there-expletive* sentences are infelicitous. They frame their story in terms of properties of determiners. A determiner D is said to be *positive strong* if $D(P)(P)$ is true for all P , and is said to be *negative strong* if $D(P)(P)$ is false for all P . A determiner which is not strong is said to be *weak*. Barwise and Cooper noted that only weak determiners are appropriate objects for *there is*. For instance, *some* and *no* are interpreted as weak determiners, because $\mathbf{some}(P)(P)$ is false if P is empty and true otherwise, and just the opposite holds for *no*. In the plural cases, *three*, *exactly two*, *many*, and *few* are weak. Quantificational plural determiners such as *most* are a bit trickier, because it is unclear whether sentences such as *most unicorns are unicorns* can be uttered truthfully if there are no unicorns. Such an utterance would seem infelicitous at best. But if the sentence is actually false, then *most* is also a weak determiner. On the other hand, **every** is positive strong. Note that if we assume conservativity, so that $D(X)(Y) \equiv D(X)(X \cap Y)$, then a determiner is strong if and only if $D(X)(\lambda z. \mathbf{true})$ holds for every X . This provides some clue as to why the quantifier $\lambda P. P(j)$, derived from the individual

j, is not an appropriate complement to *there is*.

Exercises

(7-1)

Provide a Russellian definition of the constant **the** used in a quantificational entry for the definite determiner:

$$the \Rightarrow \mathbf{the}: np \uparrow s / n.$$

It should be defined such that *the kid ran* is true if and only if there is exactly one kid and that kid ran.

(7-2)

Does the Cooper-storage approach to quantifier scope lead to the same kinds of spurious ambiguity as in non-normal natural deduction analyses?

(7-3)

Show how lexical entries of the kind illustrated in (32) can be used to produce the undesired result in (31).

(7-4)

How many readings are generated by our grammar for the following sentence, and does this match the empirical data?

- a. Every student [wrote some paper] and [read some book].
- b. Jo showed [someone every drawing] but [no one every painting].

(Hint: Try coordinating at the categories $(np \uparrow s) \setminus s$ and $(np \setminus s / np / np) \setminus np \setminus s$, respectively.)

(7-5)

(Quine 1960; Montague 1973)

Explain why our approach to the copula, which is basically that of Montague, provides a meaning for *Jo is a man* that is logically equivalent to the one suggested by Quine, namely **man(j)**.

(7-6)

Explain the distinction between the following examples with respect to the logical nature of the scope and unbounded dependency constructors.

- a. Every person that Sandy took a [picture of ___] smiled.
- b. Every person that Sandy showed [a picture] [to ___] smiled.
- c. Every person that Sandy showed [___] [a picture] smiled.

Assume that *to* is a dative-marking preposition here.

(7-7)

What are the possibilities for scoping in the following example.

- a. A visiting professor who at least one student challenged every statement of collapsed.

(7-8) (H. Hendriks 1987)
 Consider generalized quantifier categories of the form $(s/np)\backslash s$ and $s/(np\backslash s)$. Note that these categories are of the same type as $np\uparrow s$. Using these categories, determine whether it is possible to generate the correct range of scope ambiguities in the following examples.

- a. Someone likes everyone.
- b. Someone showed everyone something.

(7-9) (H. Hendriks 1987)
 Show how all of the examples in (23) can be derived, assuming that the semantic terms all result in sentences and that each argument σ_i is either a forward or backward slash, and that all of the quantifier arguments are of category $np\uparrow s$. Show that the composition of argument raising and argument lowering is simply the identity function of the appropriate type. Why does composing in the opposite order not produce the identity function?

(7-10) (Jackendoff 1972; Rooth 1985)
 The quantificational element *only*, as it occurs in the following sentences, can be analyzed as a generalized quantifier with an individual argument.

- a. Only Jo likes Morgan.

Assuming a lexical entry for *only* of category **only**: $np\uparrow s/np$, and assuming that the logical translation of this sentence is **only(j)(like(m))**, provide an appropriate definition of the constant **only**.

Jackendoff (1972) pointed out that there are truth-conditional differences which arise due to *focus*. For instance, consider the following standard examples, where the focused element is as indicated. (Hint: If the difference in truth conditions is not clear, try reading them aloud with stress on the focused element.)

- b. Jo only introduced *Brett* to Sue.
- c. Jo only introduced Brett to *Sue*.

Using the quantificational constant **only** you defined above, show how we can represent the meanings of these two sentences.

The syntax and semantics of *only* is further complicated by both polymorphism and context sensitivity, as demonstrated by the following example (Rooth 1985).

- d. Jo only ran.

Show how such an example could be treated by generalizing your definition of **only** to a generalized quantifier of type $\tau \rightarrow (\tau \rightarrow \mathbf{Bool}) \rightarrow \mathbf{Bool}$, and instantiating τ to **Ind** \rightarrow **Bool** in the above example.

As Rooth points out, even though the first argument may be determined by information such as syntactic attachment and focus, the quantification is still contextually restricted. Show how you could add an additional argument to the polymorphic **only** of type $\tau \rightarrow \mathbf{Bool}$ to provide a set of alternatives from which the second argument must be drawn. Alternatively, show how the same effect could be achieved by restricting the second argument itself.

(7-11) (Carpenter 1994a, 1994c)
 Consider the ambiguity of the following sentence.

- a. Jo broke every kid's favorite toy.

Here *Jo* may be construed as having broken the favorite toy of each kid, or just one toy, which happened to be everyone's favorite. To provide a single lexical entry that accounts for both readings, Carpenter proposed a lexical entry with the following syntactic category.

b. 's $\Rightarrow ? : (np \uparrow s) \setminus np / n$

Provide an appropriate semantic term for the possessive, which will allow for the following reading of *every kid's toy*.

c. *every kid's toy* $\Rightarrow \iota(\lambda x. \mathbf{toy}(x) \wedge \mathbf{every}(\mathbf{kid})(\lambda y. \mathbf{poss}(y)(x))) : np$

Use your lexical entry to provide both possible derivations for the ambiguous sentence above. Further, show that your lexical entry derives the lexical entry in the text with the same semantics by means of argument lowering.

(7-12)

(Keenan and Stavi 1986)

Show that possessives such as *Jo's* and *every kid's*, as we have defined them, are not permutation invariant in the sense of Definition 8 in Section 3.3.2.

Next, show that determiners composed of lexical determiners composed with adjectives are not necessarily permutation invariant. For instance, consider examples such as *every blue*, which can be analyzed as being of the category of determiners, $np \uparrow s / n$.

(7-13)

Show that if we assume n -tuple types σ^n for arbitrary types σ , then the unit type, $\mathbf{1}$, has a domain that is isomorphic to the domain of σ^0 for any type σ .

(7-14)

Assume we have a *zero type* $\mathbf{0}$, whose domain is taken to be the empty set. What does this imply for the domains $\mathbf{0} \times \sigma$, $\sigma \times \mathbf{0}$, $\mathbf{0} + \sigma$, $\sigma + \mathbf{0}$, $\mathbf{0} \rightarrow \sigma$, and $\sigma \rightarrow \mathbf{0}$?

Explain the extension of the Curry-Howard morphism to $\mathbf{0}$, under the assumption that $\mathbf{0}$ corresponds to the denotation of the propositional constant **false**.

(7-15)

Provide a category for the negative particle *not* that will account for the semantics of negated quantifiers, as in the following examples.

- Not every student studied.
- Not one student studied.
- * Not {some student / Sandy} studied.

What is the range of quantifiers that *not* can apply to? Is there a difference in readings predicted between the first example above and *every student did not study*?

(7-16)

(Sánchez Valencia 1991; Dowty 1994)

Write a grammar that accounts for the distribution of negative polarity determiners such as *any*, negative polarity adverbials such as *ever*, and negative polarity idioms such as *budge an inch*, as described in Section 3.4.

More specifically, assume that there is a binary feature on all basic syntactic categories that signifies whether or not an expression needs to be embedded inside of a negative polarity context. Thus *ever ran* should be marked as requiring a negative polarity context, but a simple verb phrase such as *ran* should be marked as not requiring a negative polarity context. Keep in mind that the need

to be embedded in a negative polarity environment should propagate, and thus not only should *any kid* be marked as requiring a negative polarity context, but so should *likes any kid* and *teacher who likes any kid*.

Negative polarity inducing elements such as the verb phrase negation *not* and the negative determiner *no* should be marked with features in such a way as to license arguments that are negative polarity. But importantly, they do not produce results which need to be embedded in negative polarity contexts. Finally, grammatical sentences will be the ones that do not require negative polarity marking.

(7-17) (Partee 1984a; Kadmon 1985)
Partee introduced the following example, which highlights the *proportion problem* for unselective binding accounts of indefinites as presented in Section 7.10.

a. Most women who own a cat are happy.
For the sake of this exercise, assume that *most* is translated as a generalized determiner binding individuals with the obvious interpretation over finite domains (see Section 8.6 for motivation).

Explain why a unselective analysis is problematic in this case. (Hint: Consider a model in which there are a few women who own a varying number of cats.)

(7-18)
Provide the lexical entries required to derive the examples in (107). Be sure to capture the difference in properties attributed to the comparative object in (107)e and (107)f.

(7-19)
Provide the derivations of *as* and *more* given in (111).
Provide the two derivations of (112)a and the one derivation of (112)b which does not violate island restrictions. (Note: the island-violating case of (112)b is also derivable because we have not enforced island constraints).

(7-20)
Provide an analysis of a complement-taking comparative, such as the following.
a. More [loyal to the home team] than Terry
Is such a scale measurable?

(7-21)
What is the meaning generated by our grammar for equatives with fractional multipliers, such as *half as tall* and *half as short*? How does this compare to that generated by *twice as tall* and *twice as short*? Be sure to discuss the interaction between the polarity of the comparative and the size of the multiplier.

(7-22)
Consider the use of *comparative free relatives*, as in the following example.
a. Sandy grew {tall / three inches tall / three inches / however tall Terry grew}.
Provide appropriate lexical assignment(s) for *grow*. Then consider how the free relative version pronoun can be assigned a lexical entry that will interact with these.

(7-23)

Show that the expressions *taller than* and *not as short as* can be assigned synonymous meanings as binary relations of category pr/np . Note that you will need to assume a category pr/pr for *not* with the same meaning as it receives as category $s/np/(s/np)$. Is there also synonymy between *shorter than* and *not as tall as*?

(7-24)

Consider the modification of number-denoting terms such as the following.

- at {least / most} five feet tall
- {exactly / approximately} 1.5 meters tall
- almost five feet tall

Provide an analysis where terms such as *at least*, *exactly*, and *almost* act as quantificational modifiers of numbers, of category $num\uparrow s/num$. Use your category to generate two readings for the following example.

- Every student is [approximately 1.5] meters tall.

Does it make any difference that we chose the category $num\uparrow s/num$ rather than $deg\uparrow s/deg$?

(7-25)

Consider the term *less*, as used in examples such as the following.

- Sandy is less tall than Terry.
- Sandy is three inches less tall than Terry.

Provide an analysis of *less* along the lines of that given for *more* in (102). (Hint: Use subtraction rather than addition in the basic semantics.) How does the term *less tall* compare to *shorter*?

(7-26)

Consider nominalized forms of comparatives, as in the following.

- Jo's height is six inches.

Extend the semantics of the copula to handle this case, and consider the type that should be assigned to *height* and the possessive. Provide a derivation.

(7-27)

(Bresnan 1973, 1975; Pollard and Sag 1994)

Bresnan and later Pollard and Sag discuss examples of nested comparison such as the following.

- This factory is [very many times / much] more productive than that one.
- Kim is much more productive than Dana.
- Kim is (three times) as much more intelligent than Sandy as Chris is more intelligent than Dana.

Can *very many times* and *much* in the first example be analyzed as a multiplier? Is there a way to extend our analysis of equatives to deal with the nesting in the second example above? (Hint: Consider a category of $s\uparrow mlt$ for the complement *Chris is ___ more intelligent than Dana*.)

(7-28)

(Gerdemann p. c.)

Under our categorization of relational nouns as taking quantified complements, show why the following example gets the wrong reading.

- picture of no one

Contrast this with a similar expression such as *sister of no one*. This suggests that the correct lexical entry for *picture* would be something along the following lines.

b. *picture* $\Rightarrow \lambda Q.\lambda x.\mathbf{pic}(x) \wedge Q(\lambda y.\mathbf{of}_2(y)(x)):n/(np\uparrow s)$

Why does this entry not produce the same undesirable form of derivation as in the entry we gave in (59)?

(7-29) (Geis 1973; Keenan and Stavi 1986; von Stechow 1993)

Consider the following *exceptive* constructions with *but*.

- a. Every student but Sandy studied.
- b. No student but Sandy studied.

Now consider the following possible lexical entry for *but*, and provide analyses of the sentences above.

c. *but* $\Rightarrow \lambda y.\lambda P.\lambda x.P(x) \wedge x \neq y:n\n/n/np$

Explain why this lexical entry does not require Sandy to be a student in either example and why it does not require Sandy to study in the second example.

von Stechow (1993) provides a semantic term for the exceptive that reduces to the following semantic behavior in the singular case, and would naturally be assigned to the following syntactic category.

d. *but* $\Rightarrow \lambda y.\lambda P.\lambda D.\lambda R.D(P - \{y\})(R) \wedge \mathbf{every}_{\text{Ind} \rightarrow \text{Bool}}(\lambda S.D(P - S)(R))$
 $(\lambda S.\{y\} \subseteq S)$

$$n \setminus (np\uparrow s/n) \setminus (np\uparrow s)/np$$

By deriving the two sentences above, explain why the semantic terms assigned to them provide the correct entailments for the studenthood and study habits of Sandy in both cases. Next, explain how the semantics above accounts for the infelicity of conjoined exception clauses (as noted by Geis 1973) and exception clauses modifying an existential quantifier.

- e. # Every kid [but Sandy and but Terry] studied.
- f. # Some kid but Terry studied.

(Hint: It may help to derive the sentences above and consider the resulting semantic assignment.)

(7-30) (Hoeksema 1987; von Stechow 1993)

Consider the following *exceptive* constructions with *but*.

- a. Every student but Sandy studied.
- b. No student but Sandy studied.

Now consider the following possible lexical entry for *but*, and provide analyses of the sentences above.

c. *but* $\Rightarrow \lambda y.\lambda P.\lambda x.P(x) \wedge x \neq y:n\n/n/np$

What is the status of the studenthood of Sandy in these examples, and is it captured by this analysis?

Does such an analysis license the following inference? (Hint: Consider the monotonicity of the quantifiers.)

- d. {Every / No} student but Sandy studied.
- e. {Every / No} student but Sandy and Terry studied.

von Stechow provides a richer analysis for the semantics of a construction such as “Det N *but* NP VP”. In the case where D is the denotation of determiner, P the property denoted by the head noun, X is the set of exceptions denoted by the NP, and R the property denoted by the verb phrase, von Stechow provides the following definition.

f. $D P \text{ but } X R \stackrel{\text{def}}{=} D(P - X)(R) \wedge \text{every}(\lambda S.D(P - S)(R))(\lambda S.C \subseteq S)$

Does this analysis solve the problems mentioned above? How could this proposal be realized syntactically in categorial grammar with a lexical entry for *but*? (Hint: Allow *but* to take all of the other components as arguments other than the verb phrase to produce a new quantifier.)

Formulate a lexical entry for *except for*, the so-called *free exceptive*, and consider how it differs from *but*. Provide a lexical entry for *except* that captures this difference.

(7-31)

Provide an analysis along the lines of that given in Section 7.12 in which gradable adjectives like *tall*, when occurring in comparative and equative constructions, are assigned meanings directly in terms of measures rather than in terms of extents. That is, *tall* would be assigned to the meaning $\lambda d.\lambda x.\text{height}(x) = d$, which involves equality rather than inequality. The burden of the extent-based reasoning must now be placed on the basic predicates in the positive construction, and on the comparative and equative expressions in those constructions.

Chapter 8

Plurals

In this chapter, we provide a type-logical account of plurality. From this perspective, the perplexing array of behaviors displayed by plural determiners, nominal modifiers, adverbials, conjunctions, and reciprocals can be classified and explained in a unified manner. We approach this task constructively, building a grammar as we motivate its various components. We conclude the chapter with a treatment of mass terms, which in many ways, closely resemble groups in their behavior.

8.1 An Ontology of Groups

The semantics of plurality has engendered a vast literature concerning the fundamental ontological organization of referential expressions in natural language. Such considerations provide a natural starting point for a type-logical account of plurality. We begin with the simple observation that plural noun phrases can be construed in more than one way, as the following examples illustrate.

- (1) a. Fifty kids sneezed. (Distributive)
b. Fifty kids gathered outside. (Collective)
c. Three kids moved the piano. (Ambiguous)

An example such as (1)a can only be true if each of the fifty kids sneezed individually. Such a construal of the subject is said to be *distributive*, because the property denoted by the verb phrase is distributed over the members of the set picked out by the subject. In contrast, (1)b can only be true if the fifty kids gathered as a group. Such readings are known as *collective*. An example such as (1)c might be used truthfully either way; three kids could have moved the piano individually, perhaps because each liked to have it in a different spot to practice, or the three kids could

have acted collectively to move the piano. We take such distinctions to be lexical properties of verbs. Some verbs, like *sneeze*, only apply to individuals. Other verbs, like *gather*, apply only to groups. Still other verbs, like *carry* can apply to both groups and individuals.

Such ambiguities between a distributive construal and collective construal of a plural noun phrase can interact with lexical properties of various predicates. Some nouns, such as *gang* and *committee*, pick out groups directly. In such cases, plural instances will induce an ambiguity when coupled with predicates that apply to groups.

- (2) a. The gang dispersed.
 b. The gangs dispersed.
 c. # The kid dispersed.
 d. # The committee sneezed.

In such cases, we can have a group predicate with a singular, as in (2)a. When such nouns occur in the plural, an ambiguity arises, as can be seen in the two interpretations of (2)b. Under a collective interpretation, the group of all the gangs dispersed, leaving open the possibility of the individual gangs remaining intact. With the distributive reading, the sentence is interpreted as stating that each of the gangs in the collection of gangs dispersed. It is important to note that whereas sets introduced by plural nouns may denote the group of which they form the members, it is not in general possible for a group to be automatically decomposed into its members. For instance, there is no reading of (2)d stating that each of the committee members sneezed. Because sneezing is a property attributable only to individuals, an infelicity arises. We consider this a pragmatic infelicity which comes about due to selectional restriction violations. Similarly, (2)c is infelicitous because dispersing is something only a group can do.

With this motivation, it is natural to seek a common domain in which to interpret individuals and groups. Following Link (1984), we simply assume that the type of groups is a subtype of the type of individuals.

(3) **Group** \subseteq **Ind**

This accounts naturally for the existence of group-denoting singular nouns like *team*, *committee*, and *group*, and of group denoting noun phrases like *Parliament*. It also allows a natural type assignment to predicates like *carry* which can apply to both individuals and groups. They will remain functions from individuals into propositions.