Type Logical Grammar

MICHAEL MOORTGAT
Abstract

CATEGORIAL GRAMMAR: a lexicalized grammar formalism based on logical type-theory. A categorial lexicon assigns one or more types to the atomic elements of a language; the assembly of form and meaning is accounted for in terms of the rules of inference for these types seen as formulas of a grammar logic. Cross-linguistic variation results from extending the invariant core of the grammar logic with facilities for structural reasoning.

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1. Background reading
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Available from http://www.let.uu.nl/~ctl/docenten/moortgat.html
2. Grammar development tools
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Richard Moot’s GRAIL system:
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- Proof-net based parser/theorem prover for multimodal type-logical grammars
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Available at http://www.let.uu.nl/~Richard.Moot/personal/grail.html
3. Constants and structural variation
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Basic questions
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Grammatical architecture
3. Constants and structural variation

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Grammatical architecture

▸ INVARIANTS: base logic
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- INVARIANTS: base logic

- VARIATION: structural module
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Grammatical architecture

▶ INVARIANTS: base logic

▶ VARIATION: structural module

▶ CONTROL: features licensing/constraining structural operations
4. Vocabulary

FORMULAS

STRUCTURES
4. Vocabulary

\[ \mathcal{F} ::= \quad \mathcal{A} \]
\[ \diamondsuit \mathcal{F} \]
\[ \Box \mathcal{F} \]
\[ \mathcal{F} \cdot \mathcal{F} \]
\[ \mathcal{F} / \mathcal{F} \]
\[ \mathcal{F} \setminus \mathcal{F} \]
4. Vocabulary

\[ \mathcal{F} ::= \mathcal{A} \quad \text{atoms} \]
\[ \Diamond \mathcal{F} \]
\[ \Box \mathcal{F} \]
\[ \mathcal{F} \cdot \mathcal{F} \]
\[ \mathcal{F} / \mathcal{F} \]
\[ \mathcal{F} \backslash \mathcal{F} \]
4. Vocabulary

\[ F ::= A \]
\[ \diamond F \]
\[ \Box F \]
\[ F \cdot F \] composition
\[ F / F \]
\[ F \setminus F \]

atoms
## 4. Vocabulary

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<td>$\mathcal{F} ::= \mathcal{A}$</td>
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<td>$\mathcal{F} \circ \mathcal{F}$</td>
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4. Vocabulary

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\mathcal{F} ::= \mathcal{A} \quad \text{atoms}
\]
\[
\Diamond \mathcal{F} \quad \text{feature checking}
\]
\[
\Box \mathcal{F} \quad \text{feature request}
\]
\[
\mathcal{F} \cdot \mathcal{F} \quad \text{composition}
\]
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\mathcal{F} / \mathcal{F} \quad \text{select right}
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4. Vocabulary

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<td>$\square \mathcal{F}$</td>
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<td>$\mathcal{F} \bullet \mathcal{F}$</td>
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<td>$S \circ S$</td>
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5. Models
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Frames $F = \langle W, R^2_\Diamond, R^3_\bullet \rangle$
5. Models

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$W$: ‘signs’, resources, expressions
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$W$: ‘signs’, resources, expressions

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$W$: ‘signs’, resources, expressions

$R^3_\bullet$: ‘Merge’, grammatical composition

$R^2_\Diamond$: ‘feature checking’, structural control

Models $\mathcal{M} = \langle F, V \rangle$
5. Models

Frames $F = \langle W, R^2_\lozenge, R^3_\bullet \rangle$

$W$: ‘signs’, resources, expressions

$R^3_\bullet$: ‘Merge’, grammatical composition

$R^2_\lozenge$: ‘feature checking’, structural control

Models $\mathcal{M} = \langle F, V \rangle$

Valuation $V : \mathcal{F} \mapsto \mathcal{P}(W)$: types as sets of expressions
6. Frames for structured expressions

\[ V(A \bullet B) = \{ z \mid \exists x \exists y. R^3 zxy \land x \in V(A) \land y \in V(B) \} \]
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\[ V(A \bullet B) = \{ z \mid \exists x \exists y. R^3 zxy \land x \in V(A) \land y \in V(B) \} \]

\( z \) expressions that can be disassembled in an \( A \) part \( x \) and a \( B \) part \( y \).
6. Frames for structured expressions

\[ V(A \cdot B) = \{ z \mid \exists x \exists y. R^3 zxy \land x \in V(A) \land y \in V(B) \} \]

\( \leadsto \) expressions \( z \) that can be disassembled in an \( A \) part \( x \) and a \( B \) part \( y \).

\[ V(\Diamond A) = \{ x \mid \exists y(R^2 xy \land y \in V(A)) \} \]
6. Frames for structured expressions

\[ V(A \bullet B) = \{ z | \exists x \exists y. R^3 zxy \& x \in V(A) \& y \in V(B) \} \]

\( \sim \) expressions \( z \) that can be disassembled in an \( A \) part \( x \) and a \( B \) part \( y \).

\[ V(\diamond A) = \{ x | \exists y. R^2 xy \& y \in V(A) \} \]

\( \sim \) expressions \( x \) obtained through feature checking from an \( A \) part \( y \).
6. Frames for structured expressions

\[ V(A \bullet B) = \{ z \mid \exists x \exists y. R^3 zxy \land x \in V(A) \land y \in V(B) \} \]

\(\rightsquigarrow\) expressions \(z\) that can be disassembled in an \(A\) part \(x\) and a \(B\) part \(y\).

\[ V(\Diamond A) = \{ x \mid \exists y. R^2 xy \land y \in V(A) \} \]

\(\rightsquigarrow\) expressions \(x\) obtained through feature checking from an \(A\) part \(y\).

Pairs of opposites. \((\Diamond, \Box), (\bullet, /)\) and \((\bullet, \backslash)\) form residuated pairs (inverse duals).
7. Invariants: the base logic
7. Invariants: the base logic

Type computation. The derivability relation is REFLEXIVE ($A \rightarrow A$) and TRANSITIVE (from $A \rightarrow B$ and $B \rightarrow C$, deduce $A \rightarrow C$).
7. **Invariants: the base logic**

**Type computation.** The derivability relation is **REFLEXIVE** \((A \rightarrow A)\) and **TRANSITIVE** (from \(A \rightarrow B\) and \(B \rightarrow C\), deduce \(A \rightarrow C\)).

Residuation laws relating pairs of opposites:
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Residuation laws relating pairs of opposites:

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\text{(RES-1)} \quad \diamond A \rightarrow B \quad \text{iff} \quad A \rightarrow \Box B
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Residuation laws relating pairs of opposites:

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\begin{align*}
\text{(RES-1)} & \quad \diamond A \rightarrow B \quad \text{iff} \quad A \rightarrow \Box B \\
\text{(RES-L)} & \quad A \bullet B \rightarrow C \quad \text{iff} \quad A \rightarrow C/B \\
\text{(RES-R)} & \quad A \bullet B \rightarrow C \quad \text{iff} \quad B \rightarrow A\setminus C
\end{align*}
\]
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(\text{RES-1}) & \quad \text{\(\text{◊} A \rightarrow B\)} & \text{iff} & & A \rightarrow \square B \\
(\text{RES-L}) & \quad A \bullet B \rightarrow C & \text{iff} & & A \rightarrow C/B \\
(\text{RES-R}) & \quad A \bullet B \rightarrow C & \text{iff} & & B \rightarrow A\backslash C
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**Completeness**
7. **Invariants: the base logic**

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Residuation laws relating pairs of opposites:

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\[(\text{RES-L}) \quad A \bullet B \rightarrow C \quad \text{iff} \quad A \rightarrow C/B\]

\[(\text{RES-R}) \quad A \bullet B \rightarrow C \quad \text{iff} \quad B \rightarrow A \setminus C\]

**Completeness**

\[A \rightarrow B \quad \text{is provable iff} \quad \forall F, V, \ V(A) \subseteq V(B)\]
8. Natural Deduction format
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Sequents $\Gamma \vdash A$: demonstration that structure $\Gamma$ has type $A$. 

8. **Natural Deduction format**

Sequents \( \Gamma \vdash A \): demonstration that structure \( \Gamma \) has type \( A \).

Axioms \( A \vdash A \).
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Sequents $\Gamma \vdash A$: demonstration that structure $\Gamma$ has type $A$.

Axioms $A \vdash A$.

Structure-building operations structural counterparts of $\diamond, \bullet$. 
8. Natural Deduction format

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Axioms $A \vdash A$.

Structure-building operations structural counterparts of ♦,●.

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See the ♦,● Introduction rules:
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\[ \Gamma \vdash A \]
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Structure-building operations structural counterparts of $\Diamond, \bullet$.

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See the $\Diamond, \bullet$ Introduction rules:

$\Gamma \vdash A \ [\Diamond I]$
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See the \( \diamond, \bullet \) Introduction rules:

\[
\frac{\Gamma \vdash A}{\langle \Gamma \rangle \vdash \diamond A} \quad [\diamond I]
\]
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**Sequents** $\Gamma \vdash A$: demonstration that structure $\Gamma$ has type $A$.

**Axioms** $A \vdash A$.

**Structure-building operations** structural counterparts of $\diamondsuit, \bullet$.

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See the $\diamondsuit, \bullet$ Introduction rules:

$$
\begin{align*}
\Gamma \vdash A & \quad \Gamma \vdash A \\
\langle \Gamma \rangle \vdash \diamondsuit A & \quad [\diamondsuit I] \\
\end{align*}
$$
8. Natural Deduction format

Sequents $\Gamma \vdash A$: demonstration that structure $\Gamma$ has type $A$.

Axioms $A \vdash A$.

Structure-building operations structural counterparts of $\diamond, \bullet$.

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See the $\diamond, \bullet$ Introduction rules:

\[
\frac{\Gamma \vdash A}{\langle \Gamma \rangle \vdash \diamond A} [\diamond I] \quad \frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma \vdash A \quad \Delta \vdash B}
\]
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See the $\Diamond, \bullet$ Introduction rules:

$$\frac{\Gamma \vdash A}{\langle \Gamma \rangle \vdash \Diamond A} \quad [\Diamond I] \quad \frac{\Gamma \vdash A \quad \Delta \vdash B}{\langle \cdot \rangle \vdash \langle \cdot \cdot \cdot \rangle \vdash \langle \cdot \cdot \cdot \rangle} \quad [\bullet I]$$
8. **Natural Deduction format**

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\[
\frac{\Gamma \vdash A}{\langle \Gamma \rangle \vdash \Diamond A} [\Diamond I] \quad \frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash A \bullet B} [\bullet I]
\]
9. Natural Deduction (cont’d)

Slash elimination
9. Natural Deduction (cont’d)

Slash elimination

\[ \Gamma \vdash A / B \]
9. Natural Deduction (cont’d)

Slash elimination

\[ \Gamma \vdash A / B \quad \Delta \vdash B \]
9. Natural Deduction (cont’d)

Slash elimination

\[ \Gamma \vdash A / B \quad \Delta \vdash B \quad [/E] \]
9. Natural Deduction (cont’d)

Slash elimination

\[
\frac{\Gamma \vdash A/B \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash A} \quad [/E]
\]
9. Natural Deduction (cont’d)

Slash elimination

\[
\frac{\Gamma \vdash A / B \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash A} \quad [/E] \quad \Gamma \vdash B
\]
9. Natural Deduction (cont’d)

Slash elimination

\[
\frac{\Gamma \vdash A/B \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash A} \quad [\text{/E}] \quad \frac{\Gamma \vdash B \quad \Delta \vdash B \setminus A}{\Gamma \circ \Delta \vdash A}
\]
9. Natural Deduction (cont’d)

Slash elimination

$$\frac{\Gamma \vdash A / B \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash A} [/E] \quad \frac{\Gamma \vdash B \quad \Delta \vdash B \setminus A}{\Gamma \circ \Delta \vdash B} [\setminus E]$$
9. **Natural Deduction (cont’d)**

Slash elimination

\[
\frac{\Gamma \vdash A/B \quad \Delta \vdash B}{\Gamma \cup \Delta \vdash A} \quad [/E] \quad \frac{\Gamma \vdash B \quad \Delta \vdash B \setminus A}{\Gamma \cup \Delta \vdash A} \quad \{/E\}
\]
9. Natural Deduction (cont’d)

Slash elimination

\[
\frac{\Gamma \vdash A/B \quad \Delta \vdash B}{\Gamma \odot \Delta \vdash A} \quad [/E] \quad \frac{\Gamma \vdash B \quad \Delta \vdash B \setminus A}{\Gamma \odot \Delta \vdash A} \quad \backslash[E]
\]

Slash introduction
9. Natural Deduction (cont’d)

Slash elimination

\[
\frac{\Gamma \vdash \frac{A}{B} \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash A} [/E] \quad \frac{\Gamma \vdash \frac{B}{\Delta} \quad \Delta \vdash B \setminus A}{\Gamma \circ \Delta \vdash A} [\setminus E]
\]

Slash introduction

\[
\Gamma \circ B \vdash A
\]
9. Natural Deduction (cont’d)

Slash elimination

\[
\frac{\Gamma \vdash A/B \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash A} \quad \text{[/E]}
\]

\[
\frac{\Gamma \vdash B \quad \Delta \vdash B\backslash A}{\Gamma \circ \Delta \vdash A} \quad \text{[/E]}
\]

Slash introduction

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\frac{}{\Gamma \circ B \vdash A} \quad \text{[/I]}
\]
9. Natural Deduction (cont’d)

Slash elimination

\[
\frac{\Gamma \vdash A \div B \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash A} \quad [/E] \quad \frac{\Gamma \vdash B \quad \Delta \vdash B \setminus A}{\Gamma \circ \Delta \vdash A} \quad [\setminus E]
\]

Slash introduction

\[
\frac{\Gamma \circ B \vdash A}{\Gamma \vdash A \div B} \quad [/I]
\]
9. **Natural Deduction (cont’d)**

**Slash elimination**

\[
\frac{\Gamma \vdash A/B \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash A} \quad [/E]
\]

\[
\frac{\Gamma \vdash B \quad \Delta \vdash B\setminus A}{\Gamma \circ \Delta \vdash A} \quad \setminus E
\]

**Slash introduction**

\[
\frac{\Gamma \circ B \vdash A}{\Gamma \vdash A/B} \quad [/I]
\]

\[
B \circ \Gamma \vdash A
\]
9. Natural Deduction (cont’d)

Slash elimination

\[
\frac{\Gamma ⊢ A/B \quad \Delta ⊢ B}{\Gamma \circ \Delta ⊢ A} \quad [/E] \quad \frac{\Gamma ⊢ B \quad \Delta ⊢ B\setminus A}{\Gamma \circ \Delta ⊢ A} \quad [\setminus E]
\]

Slash introduction

\[
\frac{\Gamma \circ B ⊢ A}{\Gamma ⊢ A/B} \quad [/I] \quad \frac{B \circ \Gamma ⊢ A}{\setminus I}
\]
9. Natural Deduction (cont’d)

Slash elimination

\[
\frac{\Gamma \vdash A/B \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash A} [/E] \quad \frac{\Gamma \vdash B \quad \Delta \vdash B \setminus A}{\Gamma \circ \Delta \vdash A} [\setminus E]
\]

Slash introduction

\[
\frac{\Gamma \circ B \vdash A}{\Gamma \vdash A/B} [/I] \quad \frac{B \circ \Gamma \vdash A}{\Gamma \vdash B \setminus A} [\setminus I]
\]
9. **Natural Deduction (cont’d)**

Slash elimination

\[
\frac{\Gamma \vdash A/B \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash A} \quad [/E] \quad \frac{\Gamma \vdash B \quad \Delta \vdash B\backslash A}{\Gamma \circ \Delta \vdash A} \quad [\backslash E]
\]

Slash introduction

\[
\frac{\Gamma \circ B \vdash A}{\Gamma \vdash A/B} \quad [/I] \quad \frac{B \circ \Gamma \vdash A}{\Gamma \vdash B\backslash A} \quad [\backslash I]
\]

9. Natural Deduction (cont’d)

Slash elimination

\[
\frac{\Gamma \vdash A / B}{\Gamma \circ \Delta \vdash A} \quad \frac{\Gamma \vdash B}{\Gamma \circ \Delta \vdash A / B}
\]

\[
\frac{\Gamma \vdash B \Delta \vdash B \setminus A}{\Gamma \circ \Delta \vdash A \setminus B}
\]

Slash introduction

\[
\frac{\Gamma \circ B \vdash A}{\Gamma \vdash A / B} \quad \frac{B \circ \Gamma \vdash A}{\Gamma \vdash B \setminus A}
\]


Features: □
9. Natural Deduction (cont’d)

Slash elimination

\[
\begin{align*}
\Gamma \vdash A / B & \quad \Delta \vdash B \quad \text{[/E]} \\
\Gamma \circ \Delta & \vdash A
\end{align*}
\]

\[
\begin{align*}
\Gamma \vdash B & \quad \Delta \vdash B \setminus A \quad \text{[\setminus E]} \\
\Gamma \circ \Delta & \vdash A
\end{align*}
\]

Slash introduction

\[
\begin{align*}
\Gamma \circ B \vdash A & \quad \text{[/I]} \\
\Gamma & \vdash A / B
\end{align*}
\]

\[
\begin{align*}
B \circ \Gamma \vdash A & \quad \text{[\setminus I]} \\
\Gamma & \vdash B \setminus A
\end{align*}
\]


Features: □

\[
\Gamma \vdash \Box A
\]
9. Natural Deduction (cont’d)

Slash elimination

\[
\frac{\Gamma \vdash A/B \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash A} \quad [/E] \quad \frac{\Gamma \vdash B \quad \Delta \vdash B \setminus A}{\Gamma \circ \Delta \vdash A} \quad [\setminus E]
\]

Slash introduction

\[
\frac{\Gamma \circ B \vdash A}{\Gamma \vdash A/B} \quad [/I] \quad \frac{B \circ \Gamma \vdash A}{\Gamma \vdash B \setminus A} \quad [\setminus I]
\]


Features: \(\Box\)

\[
\Gamma \vdash \Box A \quad [\Box E]
\]
9. Natural Deduction (cont’d)

Slash elimination

\[
\frac{\Gamma \vdash \frac{A/B}{\Delta} \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash A} \quad [/E] \quad \frac{\Gamma \vdash B \quad \Delta \vdash \frac{B/A}{\Delta}}{\Gamma \circ \Delta \vdash A} \quad [\backslash E]
\]

Slash introduction

\[
\frac{\Gamma \circ B \vdash A}{\Gamma \vdash \frac{A/B}{\Gamma}} \quad [/I] \quad \frac{B \circ \Gamma \vdash A}{\Gamma \vdash B \backslash A} \quad [\backslash I]
\]


Features: □

\[
\frac{\Gamma \vdash \Box A}{\langle \Gamma \rangle \vdash A} \quad [\Box E]
\]
9. **Natural Deduction (cont’d)**

Slash elimination

\[
\frac{\Gamma \vdash A/B \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash A} \quad \frac{\Gamma \vdash B \quad \Delta \vdash B\setminus A}{\Gamma \circ \Delta \vdash A}
\]

[/E]  \quad \[\setminus E\]

Slash introduction

\[
\frac{\Gamma \circ B \vdash A}{\Gamma \vdash A/B} \quad \frac{B \circ \Gamma \vdash A}{\Gamma \vdash B\setminus A}
\]

[/I]  \quad \[\setminus I\]


Features: □

\[
\frac{\Gamma \vdash \Box A}{\langle \Gamma \rangle \vdash A} \quad \frac{\langle \Gamma \rangle \vdash A}{\Box E}
\]

\[
\langle \Gamma \rangle \vdash A
\]
9. Natural Deduction (cont’d)

Slash elimination

\[
\frac{\Gamma \vdash A/B \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash A} \quad [/E] \quad \frac{\Gamma \vdash B \quad \Delta \vdash B\setminus A}{\Gamma \circ \Delta \vdash A} \quad [\setminus E]
\]

Slash introduction

\[
\frac{\Gamma \circ B \vdash A}{\Gamma \vdash A/B} \quad [/I] \quad \frac{B \circ \Gamma \vdash A}{\Gamma \vdash B\setminus A} \quad [\setminus I]
\]


Features: \textit{□}

\[
\frac{\Gamma \vdash \Box A}{\langle \Gamma \rangle \vdash A} \quad [\Box E] \quad \frac{\langle \Gamma \rangle \vdash A}{\Gamma \vdash A} \quad [\Box I]
\]
9. Natural Deduction (cont’d)

Slash elimination

\[ \Gamma \vdash A / B \quad \Delta \vdash B \quad \Rightarrow \quad \Gamma \circ \Delta \vdash A \quad [/E] \]

\[ \Gamma \vdash B \quad \Delta \vdash B \setminus A \quad \Rightarrow \quad \Gamma \circ \Delta \vdash A \quad [\setminus E] \]

Slash introduction

\[ \Gamma \circ B \vdash A \quad \Rightarrow \quad \Gamma \vdash A / B \quad [/I] \]

\[ B \circ \Gamma \vdash A \quad \Rightarrow \quad \Gamma \vdash B \setminus A \quad [\setminus I] \]


Features: □

\[ \Gamma \vdash \square A \quad [\square E] \]

\[ \langle \Gamma \rangle \vdash A \quad [\square I] \]
10. Sample derivation

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

the

BOTTOM UP DERIVATION: the book that Knuth wrote
10. **Sample derivation**

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

BOTTOM UP DERIVATION: the book that Knuth wrote
10.  Sample derivation

that

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

\[
\text{the} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \backslash n)/(s/np)}
\]

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

\[
\text{that} \quad \frac{(n \backslash n)/(s/np)}{\text{book}} \quad \frac{\text{the}}{n} \quad \frac{np/n}{n}
\]

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

\[
\frac{\text{book}}{n} \frac{(n \backslash n)/(s/np)}{np/n}
\]

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

\[
\begin{align*}
\text{knuth} & \quad \frac{np}{np} \\
\text{that} & \quad \frac{(n \setminus n)/(s/np)}{\text{book}} \\
\text{the} & \quad \frac{np/n}{np/n}
\end{align*}
\]

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

knuth
\[
\frac{np}{n}
\]

that
\[
\frac{(n \backslash n)/(s/np)}{\text{book}}\]

\[
\frac{\text{the}}{np/n}
\]

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

```plaintext
knuth
\frac{np}{n}

\frac{\text{book}}{n} \frac{(n \setminus n)/(s/np)}{np/n}

\text{BOTTOM UP DERIVATION: the book that Knuth wrote}
```
10. Sample derivation

\[
\begin{align*}
\text{knuth} & \quad \frac{n p}{s} / n p \\
\text{that} & \quad (n \backslash n) / (s / n p) \\
\text{the} & \quad \frac{b o o k}{n} \\
\end{align*}
\]

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

\[
\frac{\text{knuth}}{\text{np}} \quad \frac{\text{wrote}}{(\text{np} \backslash s)/\text{np}} \quad [\text{np} \vdash \text{np}]^1
\]

\[
\frac{\text{that}}{(n\backslash n)/(s/\text{np})}
\]

\[
\frac{\text{the}}{(\text{np}/n)}
\]

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

\[
\begin{align*}
\text{knuth} & \quad \frac{\text{wrote}}{(np\backslash s)/np} \quad [np \vdash np]^1 \quad [/E] \\
\text{that} & \quad \frac{\text{the book}}{(n\backslash n)/(s/np)}
\end{align*}
\]

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

\[
\begin{align*}
\text{knuth} & \quad \frac{\text{wrote}}{np} \\
\text{} & \quad \frac{(np \backslash s)/np}{np} \quad \frac{[np \vdash np]}{[\vdash E]} \\
\text{} & \quad \frac{\text{wrote} \circ np \vdash np \backslash s}{/E} \\
\text{the} & \quad \frac{\text{that}}{(n \backslash n)/(s/np)} \\
\text{book} & \quad \frac{(n \backslash n)/(s/np)}{np/n}
\end{align*}
\]

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

\[
\array{\text{knuth} \quad \frac{\text{wrote}}{np} \quad \frac{(np/s)/np \quad [np \vdash np]^1}{\text{wrote} \circ np \vdash np/s} \quad [\setminus E] \\
\, \\
\text{the} \quad \frac{\text{book}}{n} \quad \frac{(n/n)/(s np)}{(n/n)/(s np)}}
\]

\text{BOTTOM UP DERIVATION: the book that Knuth wrote}
10. Sample derivation

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

\[
\begin{align*}
\text{knuth} & \quad \frac{(np \backslash s)/np}{[np \vdash np]^1} \\
& \quad \frac{\text{wrote} \circ np \vdash np \backslash s}{[\backslash E]} \\
& \quad \frac{\text{wrote} \circ np \vdash np \backslash s}{[\backslash E]} \\
& \quad \frac{\text{knuth} \circ (\text{wrote} \circ np) \vdash s}{[? \quad \text{structural rule!}]}
\end{align*}
\]

\[
\begin{align*}
\text{the} \quad \frac{np/n}{(n/n)/(s/np)} \\
\text{book} \quad \frac{n}{(n/n)/(s/np)} \\
\text{that} \quad \frac{n}{(n/n)/(s/np)}
\end{align*}
\]

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

\[
\begin{array}{c}
\text{the} \quad \frac{(n/n)/(s/np)}{np/n} \\
\text{book} \quad \frac{(n/n)/(s/np)}{np/n} \\
\text{that} \quad \frac{(n/n)/(s/np)}{np/n} \\
\text{wrote} \quad \frac{(np\backslash s)/np}{np} \quad [np \vdash np]^1 \quad [\backslash E] \\
\text{wrote} \circ np \vdash np\backslash s \\
\text{knuth} \quad \frac{(np\backslash s)/np}{np} \quad [\backslash E] \\
\text{knuth} \circ (\text{wrote} \circ np) \vdash s \\
\text{knuth} \circ (\text{wrote} \circ np) \circ np \vdash s \\
\text{? — structural rule!} 
\end{array}
\]

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

**bottom up derivation:** the book that Knuth wrote
10. Sample derivation

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

BOTTOM UP DERIVATION: the book that Knuth wrote
10. Sample derivation

\[
\begin{align*}
\text{the} \quad & \text{that} \quad \text{book} \quad \text{wrote} \\
\frac{np/n \quad (n\n)/(s/np)}{\text{bottom up derivation: the book that Knuth wrote}}
\end{align*}
\]
11. Sample derivation (cont’d)

TOP DOWN DERIVATION: the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\text{the } \circ (\text{book } \circ (\text{that } \circ (\text{knuth } \circ \text{wrote})))) \vdash np
\]

**TOP DOWN DERIVATION:** the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\text{the } (\text{book } (\text{that } (\text{knuth } \text{wrote})))) \vdash np \quad [/E]
\[
\text{TOP DOWN DERIVATION: the book that Knuth wrote}
\]
11. Sample derivation (cont’d)

\[
\frac{np/n}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))))} \vdash np \quad [/E]
\]

TOP DOWN DERIVATION: the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\begin{align*}
np / n \\
\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})))) & \vdash np \quad [\!/E]
\end{align*}
\]

TOP DOWN DERIVATION: the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\begin{align*}
\text{the} & \quad \frac{np/n}{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))))} \vdash np \quad [/E]
\end{align*}
\]

TOP DOWN DERIVATION: the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\begin{align*}
\text{the} & \quad (np/n) \\
\frac{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))}{\equiv n} & \quad \vdash n \\
\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})))) & \equiv np \\
& \quad \vdash np \\
& \quad [/E]
\end{align*}
\]

TOP DOWN DERIVATION: the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\begin{align*}
\text{the} & \\
\text{\(np/n\)} & \quad \text{book \(\circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))\)} \vdash n \quad [\backslash E] \\
\text{the} & \quad \text{\(\circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))))\)} \vdash np \quad [/E]
\end{align*}
\]  

**TOP DOWN DERIVATION:** the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\begin{align*}
\text{the} & \quad \quad n \\
np/n & \quad \quad \text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n \quad [\backslash E] \\
\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) & \vdash np \quad [/E]
\end{align*}
\]

TOP DOWN DERIVATION: the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\begin{align*}
\text{the} & \quad n \\
np/n & \quad \text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n \\
\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) & \vdash np
\end{align*}
\]

\[\text{[}/E]\]

TOP DOWN DERIVATION: the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\text{\begin{array}{c}
\frac{\text{the} \quad \text{book}}{n} \\
\frac{\text{np} / n}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} \quad [\backslash E]
\end{array}}
\]

\[
\text{\begin{array}{c}
\frac{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})))) \vdash np} \quad [/ E]
\end{array}}
\]

TOP DOWN DERIVATION: the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\frac{\text{the}}{np/n} \quad \frac{\text{book}}{n} \quad \frac{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n \setminus n}{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n} \quad [\setminus E] \\
\frac{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})))) \vdash np}{[/E]}
\]

TOP DOWN DERIVATION: the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\begin{align*}
\text{NP/}\text{N} & \quad \text{book} \quad \frac{\text{that} \circ (\text{knuth} \circ \text{wrote})}{\vdash \text{N}} \\ 
\text{n} & \quad \frac{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))}{\vdash \text{N}} \\ 
\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) & \vdash \text{NP} \\
\end{align*}
\]

\text{TOP DOWN DERIVATION: the book that Knuth wrote}
11. Sample derivation (cont’d)

\[
\begin{align*}
\text{the} & \quad \text{book} \quad \frac{(n \setminus n)/(s/np)}{np/n} \quad \frac{\text{that} \circ (\text{knuth} \circ \text{wrote})}{n \setminus n} \quad \frac{\text{\[\text{// \ E}\]}}{[\\text{// \ E}]}
\end{align*}
\]

TOP DOWN DERIVATION: the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\begin{align*}
\text{TOP DOWN DERIVATION: } & \text{the book that Knuth wrote} \\
\frac{\text{the}}{np/n} & \frac{\text{book}}{n} \frac{(n \backslash n)/(s/np)}{\text{that } \circ (\text{knuth } \circ \text{wrote})} \vdash n \backslash n &[/E] \\
\frac{\text{np}/n} & \frac{\text{book } \circ (\text{that } \circ (\text{knuth } \circ \text{wrote}))} {n} \vdash n &[\backslash E] \\
\frac{\text{the } \circ (\text{book } \circ (\text{that } \circ (\text{knuth } \circ \text{wrote}))))} {np} \vdash np &[/E]
\end{align*}
\]
11. Sample derivation (cont’d)

\[
\begin{align*}
\text{the} & \quad \frac{\text{that}}{(n\backslash n)/(s/np)} \quad \frac{(n\backslash n)/(s/np) \vdash n\backslash n}{[\backslash E]} \\
np/n & \quad \frac{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n \backslash n}{[\backslash E]} \\
\text{the} & \quad \frac{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash np}{[\backslash E]}
\end{align*}
\]

**TOP DOWN DERIVATION:** the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\begin{align*}
\text{the} & \quad \text{np/n} \\
\text{book} & \quad \frac{(n\, n)/(s\, np)}{\text{that}} \\
& \quad \text{knuth} \circ \text{wrote} \vdash s\, np \\
\frac{(\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n\, n}{[\setminus E]} \\
\frac{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n}{[\setminus E]} \\
\frac{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})))) \vdash np}{[/E]} \\
\end{align*}
\]

TOP DOWN DERIVATION: the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\begin{align*}
\text{the} & \frac{n}{np/n} \quad \frac{\text{book}}{(n\backslash n)/(s/np)} \quad \frac{\text{that}}{\text{that} \circ (\text{knuth} \circ \text{wrote})} & \frac{\text{knuth} \circ \text{wrote} \vdash s/np}{\vdash n\backslash n} & \frac{\vdash n\backslash n}{\vdash n} & \text{top down derivation: the book that Knuth wrote} \\
\text{np/n} & \frac{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n}{\vdash np}
\end{align*}
\]
11. Sample derivation (cont’d)

\[
\begin{align*}
\text{top down derivation: the book that Knuth wrote} \\
\text{the} & \quad \text{book} \quad \frac{\text{that}}{(n\backslash n)/(s/np)} \quad \frac{(\text{knuth} \circ \text{wrote}) \circ np \vdash s}{[/I]^1} \\
np/n & \quad n \quad \frac{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n\backslash n}{[/E]} \\
np/n & \quad \text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n \quad [/E] \\
\text{the} & \quad (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np \quad [/E]
\end{align*}
\]
11. Sample derivation (cont’d)

\[
\begin{align*}
\frac{\text{the}}{\text{the book}} & \quad \frac{\text{that}}{\text{that} \circ (\text{k}\text{nuth} \circ \text{wrote})} & \quad \frac{\text{that} \circ (\text{k}\text{nuth} \circ \text{wrote}) \circ \text{np} \vdash s}{\text{structural rule}} \\
\frac{\text{np/n}}{(n \backslash n)/(s/\text{np})} & \quad \frac{\text{knuth} \circ \text{wrote} \vdash s/\text{np}}{\text{[/I]}^1} & \quad \frac{\text{knuth} \circ \text{wrote} \vdash s/\text{np}}{\text{[/E]}} \\
\frac{\text{book} \circ (\text{that} \circ (\text{k}\text{nuth} \circ \text{wrote})) \vdash n \backslash n}{\text{[/E]}} & \quad \frac{\text{book} \circ (\text{that} \circ (\text{k}\text{nuth} \circ \text{wrote})) \vdash n \backslash n}{\text{[/E]}} \\
\frac{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{k}\text{nuth} \circ \text{wrote}))) \vdash \text{np}}{\text{[/E]}}
\end{align*}
\]

**TOP DOWN DERIVATION:** the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\begin{align*}
\text{the} & \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{\frac{\text{that} \circ (\text{knuth} \circ \text{wrote})}{\frac{\text{n}}{\text{n}}} \vdash \frac{\frac{(\text{n} \setminus \text{n})/\left(\text{s}/\text{np}\right)}{\text{knuth} \circ (\text{wrote} \circ \text{np})} \vdash \text{s}}{\text{structural rule}}} \quad \frac{\text{knuth} \circ \text{wrote} \vdash \frac{\text{s}/\text{np}}{\text{I}}} \quad \frac{\vdash \frac{\text{n} \setminus \text{n}}{\text{E}}} \quad \frac{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash \frac{\text{np}}{\text{E}}} \quad \frac{\text{TOP DOWN DERIVATION: the book that Knuth wrote}}{\text{First} \quad \text{Last} \quad \text{Prev} \quad \text{Next}}
\end{align*}
\]
11. Sample derivation (cont’d)

**TOP DOWN DERIVATION:** the book that Knuth wrote

\[
\text{the} \quad \frac{\text{book}}{n} \quad \frac{\text{that}}{(n \setminus n)/(s/np)} \quad \frac{\text{that} \circ \text{(knuth} \circ \text{wrote})}{n \setminus n} \quad \frac{\text{knuth} \circ \text{(wrote} \circ np)}{s} \quad \frac{[\setminus E]}{[\text{structural rule}]} \quad \frac{\text{(knuth} \circ \text{wrote}) \circ np}{s} \quad \frac{[/I]^1}{[/E]} \quad \frac{\text{knuth} \circ \text{wrote}}{s/np} \quad \frac{[\setminus E]}{[/E]} \quad \frac{\text{book} \circ \text{(that} \circ \text{(knuth} \circ \text{wrote}))}{n} \quad \frac{[\setminus E]}{[/E]} \quad \frac{\text{the} \circ \text{(book} \circ \text{(that} \circ \text{(knuth} \circ \text{wrote}))}}{np} \quad \frac{[\setminus E]}{[/E]}
\]
11. Sample derivation (cont’d)

\[
\frac{\text{that}}{(n\setminus n)/(s/np)} \quad \frac{\text{np}}{\text{knuth} \circ (\text{wrote} \circ np) \vdash s} \quad \frac{\text{knuth} \circ \text{wrote} \vdash s/np}{\text{structural rule}} \quad [\setminus E] \\
\frac{\text{book}}{\text{n}} \quad \frac{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n\setminus n}{[\setminus E]} \\
\frac{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n}{[\setminus E]} \\
\frac{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np}{[\setminus E]}
\]

TOP DOWN DERIVATION: the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\begin{align*}
\frac{\text{the}}{np/n} & \quad np \\
\frac{\text{book}}{n} & \quad \text{that} \\
\frac{(n\setminus n)/\left(s/np\right)}{\text{(knuth \circ wrote) \circ np} \vdash s} & \quad \text{[}\setminus E\text{]} \\
\frac{\text{knuth \circ wrote}}{s/np} & \quad \text{[structural rule]} \\
\frac{\text{[}/I\text{]}^1}{\vdash s} & \quad \text{[}/E\text{]} \\
\frac{\text{the \circ (book \circ (that \circ (knuth \circ wrote)))}}{n} & \quad \text{[}/E\text{]} \\
\frac{\text{the \circ (book \circ (that \circ (knuth \circ wrote)))}}{np} & \quad \text{[}/E\text{]} \\
\end{align*}
\]

TOP DOWN DERIVATION: the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\begin{align*}
\text{the} & \quad \text{np/n} \\
\text{np/that} & \quad \text{(n/n)/(s/np)} \\
\text{book/that} & \quad \text{knuth/o (wrote/o np)} \vdash s \\
\text{book/o (that/o (knuth/o wrote))} & \quad \text{knuth/o wrote} \vdash s/np \\
\text{book/o (that/o (knuth/o wrote))} & \quad \text{np} \\
\end{align*}
\]

\text{TOP DOWN DERIVATION: the book that Knuth wrote}
11. Sample derivation (cont’d)

\[
\begin{align*}
\text{the} & \quad \frac{n}{np/n} \quad \frac{\text{book}}{(n\backslash n)/(s/np)} \quad \frac{\text{that}}{knuth} \quad \frac{\text{wrote} \circ np \vdash np\backslash s}{[\backslash E]} \quad \frac{knuth \circ (\text{wrote} \circ np) \vdash s}{[\text{structural rule}]} \\
& \quad \frac{\text{that} \circ (knuth \circ wrote) \vdash n\backslash n}{[\backslash E]} \quad \frac{knuth \circ wrote \vdash s/np}{/[I]^1} \quad \frac{knuth \circ wrote \vdash s/np}{/[E]} \\
& \quad \frac{\text{the} \circ (\text{book} \circ (\text{that} \circ (knuth \circ wrote))) \vdash n}{[\backslash E]} \quad \frac{\text{the} \circ (\text{book} \circ (\text{that} \circ (knuth \circ wrote))) \vdash np}{[\backslash E]}
\end{align*}
\]

TOP DOWN DERIVATION: the book that Knuth wrote
11. Sample derivation (cont’d)

**TOP DOWN DERIVATION:** the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\begin{align*}
\text{knuth} & \quad \frac{(np\backslash s)/np}{np} \\
\text{wrote} \circ np & \vdash np\backslash s \\
\frac{\text{knuth} \circ (\text{wrote} \circ np) \vdash s}{[\backslash E]} \\
\frac{(\text{knuth} \circ \text{wrote}) \circ np \vdash s}{[\text{structural rule}]} \\
\frac{\text{knuth} \circ \text{wrote} \vdash s/np}{[\backslash I]^1} \\
\frac{\text{that} \circ (\text{knuth} \circ \text{wrote}) \vdash n\backslash n}{[\backslash E]} \\
\frac{\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote})) \vdash n\backslash n}{[\backslash E]} \\
\frac{\text{the} \circ (\text{book} \circ (\text{that} \circ (\text{knuth} \circ \text{wrote}))) \vdash np}{[\backslash E]}
\end{align*}
\]

TOP DOWN DERIVATION: the book that Knuth wrote
11. Sample derivation (cont’d)

$$\text{knuth} \frac{(np\backslash s)/np}{\text{wrote} \circ np \vdash np\backslash s} [/E]$$

$$\text{knuth} \circ (\text{wrote} \circ np) \vdash s [/E]$$

$$\frac{(knuth \circ wrote) \circ np \vdash s}{\text{structural rule}}$$

$$\text{knuth} \circ wrote \vdash s/np [/I]^1$$

$$\frac{\text{the} \circ (knuth \circ wrote) \vdash n \backslash n}{\text{structural rule}}$$

$$\frac{(n\backslash n)/(s/np)}{(n\backslash n)/((s/np)/(np/n))}$$

$$\frac{\text{book} \circ (knuth \circ wrote) \vdash s/np}{\text{structural rule}}$$

TOP DOWN DERIVATION: the book that Knuth wrote
11. Sample derivation (cont’d)

\[
\begin{align*}
&\text{the} \quad \frac{\text{book}}{np/n} \quad \frac{\text{that}}{(n\backslash n)/(s/np)} \quad \frac{\text{wrote}}{(np\backslash s)/np} \\
&\text{knuth} \quad np \quad \frac{wrote \circ np \vdash np\backslash s}{\backslash E} \\
&\text{knuth} \circ (wrote \circ np) \vdash s \quad \text{[structural rule]} \\
&\text{knuth} \circ wrote \vdash s/np \quad \text{[/I]}^1 \\
&\text{that} \circ (knuth \circ wrote) \vdash n\backslash n \quad \text{[/E]} \\
&\text{book} \circ (\text{that} \circ (knuth \circ wrote)) \vdash n \quad \text{[/E]} \\
&\text{the} \circ (\text{book} \circ (\text{that} \circ (knuth \circ wrote))) \vdash np \quad \text{[/E]} \\
\end{align*}
\]

**TOP DOWN DERIVATION:** the book that Knuth wrote
11. Sample derivation (cont’d)

TOP DOWN DERIVATION: the book that Knuth wrote
12. Illustration: solving type equations
12. Illustration: solving type equations

LEXICON: type assignment relation
12. Illustration: solving type equations

LEXICON: type assignment relation

LEARNING new words: solving for unknowns.
12. **Illustration: solving type equations**

LEXICON: type assignment relation

LEARNING new words: solving for unknowns.

**Example.** Given Alice ⊢ np, and Alice slaapt ⊢ s (‘Alice sleeps’)
12. Illustration: solving type equations

LEXICON: type assignment relation

LEARNING new words: solving for unknowns.

Example. Given Alice ⊢ np, and Alice slaapt ⊢ s (‘Alice sleeps’)

\[ np \bullet slaapt \vdash s \]
12. **Illustration: solving type equations**

LEXICON: type assignment relation

LEARNING new words: solving for unknowns.

**Example.** Given Alice ⊢ np, and Alice slaapt ⊢ s (‘Alice sleeps’)

\[
\begin{align*}
np \bullet slaapt & \vdash s \\
slaapt & \vdash np\backslash s
\end{align*}
\]
12. Illustration: solving type equations

LEXICON: type assignment relation

LEARNING new words: solving for unknowns.

Example. Given Alice ⊨ np, and Alice slaapt ⊨ s (‘Alice sleeps’)

\[ np \bullet slaapt \quad ⊨ \quad s \]
\[ slaapt \quad ⊨ \quad np\backslash s \]

Example. Alice slaapt rustig (‘Alice sleeps quietly’)
12. Illustration: solving type equations

LEXICON: type assignment relation

LEARNING new words: solving for unknowns.

Example. Given Alice ⊢ np, and Alice slaapt ⊢ s (‘Alice sleeps’)

\[
np \bullet slaapt \vdash s \\
slaapt \vdash np \setminus s
\]

Example. Alice slaapt rustig (‘Alice sleeps quietly’)

\[
np \bullet (np \setminus s \bullet rustig) \vdash s
\]
12. **Illustration: solving type equations**

LEXICON: type assignment relation

LEARNING new words: solving for unknowns.

**Example.** Given Alice ⊨ np, and Alice slaapt ⊨ s (‘Alice sleeps’)

\[
np \bullet slaapt \ ⊨ \ s \\
slaapt \ ⊨ \ np\backslash s
\]

**Example.** Alice slaapt rustig (‘Alice sleeps quietly’)

\[
np \bullet (np\backslash s \bullet rustig) \ ⊨ \ s \\
np\backslash s \bullet rustig \ ⊨ \ np\backslash s
\]
12. Illustration: solving type equations

LEXICON: type assignment relation
LEARNING new words: solving for unknowns.

Example. Given Alice ⊢ np, and Alice slaapt ⊢ s (‘Alice sleeps’)

\[
\begin{align*}
np \bullet slaapt & \vdash s \\
slaapt & \vdash np \backslash s
\end{align*}
\]

Example. Alice slaapt rustig (‘Alice sleeps quietly’)

\[
\begin{align*}
np \bullet (np \backslash s \bullet rustig) & \vdash s \\
np \backslash s \bullet rustig & \vdash np \backslash s \\
rustig & \vdash (np \backslash s) \backslash (np \backslash s)
\end{align*}
\]
13. Exercise
13. Exercise

Transform CFG rules in categorial type assignments.
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Transform CFG rules in categorial type assignments.

Strategy:

- Assign a constituent structure bracketing
13. Exercise

Transform CFG rules in categorial type assignments.

Strategy:

- Assign a constituent structure bracketing
- Impose a function/argument articulation within each constituent domain
14. Exercise
14. Exercise

Subtyping through feature decorations:
14. Exercise

Subtyping through feature decorations:

\[ \Diamond \Box A \rightarrow A \rightarrow \Box \Diamond A \]
14.  Exercise

Subtyping through feature decorations:

\[ \Diamond \Box A \rightarrow A \rightarrow \Box \Diamond A \]

Applications agreement patterns, scope construal, licensing/antilicensing relations, polarity sensitivity.
14. Exercise

Subtyping through feature decorations:

\[\Diamond \Box A \rightarrow A \rightarrow \Box \Diamond A\]

Applications agreement patterns, scope construal, licensing/antilicensing relations, polarity sensitivity.


Visit

15. Variation: the structural module
15.  Variation: the structural module

Diversity invariant BASE MODULE + variable STRUCTURAL PLUG-INS
15. **Variation: the structural module**

Diversity invariant BASE MODULE + variable STRUCTURAL PLUG-INS

Structural module non-logical axioms, POSTULATES.
15. Variation: the structural module

Diversity invariant BASE MODULE + variable STRUCTURAL PLUG-INS

Structural module non-logical axioms, POSTULATES.

Global versus controlled Structural rules can be introduced in two ways:
15. Variation: the structural module

Diversity invariant BASE MODULE + variable STRUCTURAL PLUG-INS

Structural module non-logical axioms, POSTULATES.

Global versus controlled Structural rules can be introduced in two ways:

- GLOBAL: too crude — destroys grammatically relevant information.
15. *Variation: the structural module*

Diversity invariant BASE MODULE + variable STRUCTURAL PLUG-INS

Structural module non-logical axioms, POSTULATES.

Global versus controlled Structural rules can be introduced in two ways:

- **GLOBAL:** too crude — destroys grammatically relevant information.
- **CONTROLLED:** use ♦, □ as licensing/blocking features.
15. Variation: the structural module

Diversity invariant BASE MODULE + variable STRUCTURAL PLUG-INS

Structural module non-logical axioms, POSTULATES.

Global versus controlled Structural rules can be introduced in two ways:

▶ GLOBAL: too crude — destroys grammatically relevant information.
▶ CONTROLLED: use ♦, □ as licensing/blocking features.

Compare Global versus controlled reordering.
15. Variation: the structural module

Diversity invariant BASE MODULE + variable STRUCTURAL PLUG-INS

Structural module non-logical axioms, POSTULATES.

Global versus controlled Structural rules can be introduced in two ways:

▶ GLOBAL: too crude — destroys grammatically relevant information.
▶ CONTROLLED: use ♦, □ as licensing/blocking features.

Compare Global versus controlled reordering.

\[ A \bullet B \rightarrow B \bullet A \]

\[ \Diamond A \bullet B \rightarrow B \bullet \Diamond A \]
16. **Frame constraints**

Non-logical axioms each structural postulate comes with a *constraint* on the interpretation of the Merge/feature checking relations.
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*Non-logical axioms* each structural postulate comes with a *constraint* on the interpretation of the Merge/feature checking relations.

Example
16. Frame constraints

Non-logical axioms each structural postulate comes with a *constraint* on the interpretation of the Merge/feature checking relations.

Example

\[ \Diamond A \bullet (B \bullet C) \rightarrow (\Diamond A \bullet B) \bullet C \]
16. Frame constraints

Non-logical axioms each structural postulate comes with a constraint on the interpretation of the Merge/feature checking relations.

Example

\[ \Diamond A \bullet (B \bullet C) \rightarrow (\Diamond A \bullet B) \bullet C \]
17. Multidimensional composition
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Composition modes  Different families of operators can be put together in one grammar:
17. Multidimensional composition

Composition modes  Different families of operators can be put together in one grammar:

\[ \diamond_j A \vdash B \iff A \vdash \square_j B \]

\[ A \vdash C/_{i}B \iff A \bullet_{i} B \vdash C \iff B \vdash A\setminus_{i}C \]
17. Multidimensional composition

Composition modes  Different families of operators can be put together in one grammar:

\[ \text{\textbullet}_j A \vdash B \iff A \vdash \text{\textbullet}_j B \]
\[ A \vdash C \text{\textbar}_i B \iff A \cdot_i B \vdash C \iff B \vdash A \backslash_i C \]

Expressive possibilities
17. Multidimensional composition

Composition modes  Different families of operators can be put together in one grammar:

\[ \Diamond_j A \vdash B \iff A \vdash \Box_j B \]

\[ A \vdash C/_{i}B \iff A \bullet_i B \vdash C \iff B \vdash A\backslash_i C \]

Expressive possibilities

- the LOGICAL rules for each family are invariant
17. Multidimensional composition

Composition modes  Different families of operators can be put together in one grammar:

\[ \diamond_j A \vdash B \iff A \vdash \Box_j B \]
\[ A \vdash C /_i B \iff A \bullet_i B \vdash C \iff B \vdash A \backslash_i C \]

Expressive possibilities

- the LOGICAL rules for each family are invariant
- STRUCTURAL rules may be different
17. Multidimensional composition

Composition modes Different families of operators can be put together in one grammar:

\[ \diamond_j A \vdash B \quad \text{iff} \quad A \vdash \Box_j B \]
\[ A \vdash C/_{i} B \quad \text{iff} \quad A \bullet_{i} B \vdash C \quad \text{iff} \quad B \vdash A\backslash_{i} C \]

Expressive possibilities

- the LOGICAL rules for each family are invariant
- STRUCTURAL rules may be different
- opportunity for INTERACTION postulates
17. Multidimensional composition

Composition modes  Different families of operators can be put together in one grammar:

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Expressive possibilities

- the LOGICAL rules for each family are invariant
- STRUCTURAL rules may be different
- opportunity for INTERACTION postulates

Example  Word grammar versus phrase grammar.
18. Exercise

Controlling semi-free word order in Latin.
19. Complexity issues
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Constraints on structural postulates and their correlates for computational complexity:
19. Complexity issues

Constraints on structural postulates and their correlates for computational complexity:

**linearity constraint** context-sensitive ceiling, PSPACE (Moot 2002)
19. Complexity issues

Constraints on structural postulates and their correlates for computational complexity:

**linearity constraint** context-sensitive ceiling, PSPACE (Moot 2002)

**sublinear tightening** LTAG embedding; polynomial (Moot 2002)
19. Complexity issues

Constraints on structural postulates and their correlates for computational complexity:

- **linearity constraint** context-sensitive ceiling, PSPACE (Moot 2002)
- **sublinear tightening** LTAG embedding; polynomial (Moot 2002)
- **losing track of occurrences** Turing expressivity (Lambek 1993, Carpenter 1999)
19. **Complexity issues**

Constraints on structural postulates and their correlates for computational complexity:

*linearity constraint* context-sensitive ceiling, PSPACE (Moot 2002)

*sublinear tightening* LTAG embedding; polynomial (Moot 2002)

*losing track of occurrences* Turing expressivity (Lambek 1993, Carpenter 1999)

20. Structural patterns
20. **Structural patterns**

Consider the left/right asymmetry of the distributivity patterns below.
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\[ \diamond A \bullet (B \bullet C) \leftrightarrow (\diamond A \bullet B) \bullet C \quad (Pl1) \]
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Consider the left/right asymmetry of the distributivity patterns below.

\[ \diamond A \bullet (B \bullet C) \leftrightarrow (\diamond A \bullet B) \bullet C \quad (Pl1) \]
\[ \diamond A \bullet (B \bullet C) \leftrightarrow B \bullet (\diamond A \bullet C) \quad (Pl2) \]
20. **Structural patterns**

Consider the left/right asymmetry of the distributivity patterns below.

\[ \Diamond A \cdot (B \cdot C) \leftrightarrow (\Diamond A \cdot B) \cdot C \quad (Pl1) \]
\[ \Diamond A \cdot (B \cdot C) \leftrightarrow B \cdot (\Diamond A \cdot C) \quad (Pl2) \]
\[ (A \cdot B) \cdot \Diamond C \leftrightarrow (A \cdot \Diamond C) \cdot B \quad (Pr2) \]
20. **Structural patterns**

Consider the left/right asymmetry of the distributivity patterns below.

\[ \diamond A \bullet (B \bullet C) \leftrightarrow (\diamond A \bullet B) \bullet C \quad (Pl1) \]
\[ \diamond A \bullet (B \bullet C) \leftrightarrow B \bullet (\diamond A \bullet C) \quad (Pl2) \]
\[ (A \bullet B) \bullet \diamond C \leftrightarrow (A \bullet \diamond C') \bullet B \quad (Pr2) \]
\[ (A \bullet B) \bullet \diamond C \leftrightarrow A \bullet (B \bullet \diamond C') \quad (Pr1) \]
20. **Structural patterns**

Consider the left/right asymmetry of the distributivity patterns below.

\[ \diamond A \bullet (B \bullet C) \leftrightarrow (\diamond A \bullet B) \bullet C \quad (Pl1) \]
\[ \diamond A \bullet (B \bullet C) \leftrightarrow B \bullet (\diamond A \bullet C) \quad (Pl2) \]

\[ (A \bullet B) \bullet \diamond C \leftrightarrow (A \bullet \diamond C') \bullet B \quad (Pr2) \]
\[ (A \bullet B) \bullet \diamond C \leftrightarrow A \bullet (B \bullet \diamond C') \quad (Pr1) \]

**Output → Input:** revealing a feature-marked constituent
20. Structural patterns

Consider the left/right asymmetry of the distributivity patterns below.

\[ \diamond A \ast (B \ast C) \leftrightarrow (\diamond A \ast B) \ast C \quad (Pl1) \]
\[ \diamond A \ast (B \ast C) \leftrightarrow B \ast (\diamond A \ast C) \quad (Pl2) \]
\[ (A \ast B) \ast \diamond C \leftrightarrow (A \ast \diamond C') \ast B \quad (Pr2) \]
\[ (A \ast B) \ast \diamond C \leftrightarrow A \ast (B \ast \diamond C') \quad (Pr1) \]

Output $\rightarrow$ Input: revealing a feature-marked constituent

Input $\leftarrow$ Output: hiding a marked constituent
20. **Structural patterns**

Consider the left/right asymmetry of the distributivity patterns below.

\[ \diamond A \bullet (B \bullet C) \leftrightarrow (\diamond A \bullet B) \bullet C \quad (Pl1) \]
\[ \diamond A \bullet (B \bullet C) \leftrightarrow B \bullet (\diamond A \bullet C) \quad (Pl2) \]
\[ (A \bullet B) \bullet \diamond C \leftrightarrow (A \bullet \diamond C') \bullet B \quad (Pr2) \]
\[ (A \bullet B) \bullet \diamond C \leftrightarrow A \bullet (B \bullet \diamond C') \quad (Pr1) \]

**Output → Input:** revealing a feature-marked constituent

**Input ← Output:** hiding a marked constituent

**Pl1, Pl2:** extending right selection (/)
20. **Structural patterns**

Consider the left/right asymmetry of the distributivity patterns below.

\[
\begin{align*}
\Diamond A \bullet (B \bullet C) & \leftrightarrow (\Diamond A \bullet B) \bullet C \quad (Pl1) \\
\Diamond A \bullet (B \bullet C) & \leftrightarrow B \bullet (\Diamond A \bullet C) \quad (Pl2) \\
(A \bullet B) \bullet \Diamond C & \leftrightarrow (A \bullet \Diamond C') \bullet B \quad (Pr2) \\
(A \bullet B) \bullet \Diamond C & \leftrightarrow A \bullet (B \bullet \Diamond C') \quad (Pr1)
\end{align*}
\]

**Output → Input**: revealing a feature-marked constituent

**Input ← Output**: hiding a marked constituent

*Pl1, Pl2*: extending right selection (/)

*Pr1, Pr2*: extending left selection (\)
21. Typological motivation
21. **Typological motivation**

Correlates between left/right distributivity patterns and typological distinction: head-initial versus head-final phrases/languages.
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Correlates between left/right distributivity patterns and typological distinction: head-initial versus head-final phrases/languages.

**Case studies**
21. Typological motivation

Correlates between left/right distributivity patterns and typological distinction: head-initial versus head-final phrases/languages.

Case studies

▶ Extraction
21. Typological motivation

Correlates between left/right distributivity patterns and typological distinction: head-initial versus head-final phrases/languages.

Case studies

- Extraction
- Head-adjunction: verb-clusters, cliticization
21. Typological motivation

Correlates between left/right distributivity patterns and typological distinction: head-initial versus head-final phrases/languages.

Case studies

▶ Extraction

▶ Head-adjunction: verb-clusters, cliticization

Further exploration Moortgat 1999a and 1999b, and the fragments in
http://www.let.uu.nl/~Michael.Moortgat/personal/Courses/fragments/
22. Comparison
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Compare Minimalist Grammars (MG) and Type Logical Grammars (TLG).
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Similarities
22. **Comparison**

Compare Minimalist Grammars (MG) and Type Logical Grammars (TLG).

**Similarities**

- **Lexicalized frameworks**
22. **Comparison**

Compare Minimalist Grammars (MG) and Type Logical Grammars (TLG).

**Similarities**

- **LEXICALIZED frameworks**
- **DERIVATIONAL view of composition**
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Compare Minimalist Grammars (MG) and Type Logical Grammars (TLG).

Similarities

▸ Lexicalized frameworks

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Differences
22. Comparison

Compare Minimalist Grammars (MG) and Type Logical Grammars (TLG).

Similarities

▶ Lexicalized frameworks
▶ Derivational view of composition

Differences

▶ Structural reasoning: MG hard-wired (‘Move’), TLG flexible
22. **Comparison**

Compare Minimalist Grammars (MG) and Type Logical Grammars (TLG).

**Similarities**

- **LEXICALIZED** frameworks
- **DERIVATIONAL** view of composition

**Differences**

- Structural reasoning: MG hard-wired (‘Move’), TLG flexible
- Higher order types, hypothetical reasoning: TLG yes, MG no.
22. **Comparison**

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**Similarities**

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