CHAPTER 6

Quantifier Dynamics

Let us take stock of what we have achieved so far. In the last two chapters, we have focused on what is demanded of a dynamic semantic theory of discourse anaphora in accounting for some aspects of plurality and quantification. We saw the need for a structured notion of context and argued that careful variable management is in order. So far, we have restricted our attention to distributivity (in particular, by focusing on the floating distributive quantifier 'each'), while the noun phrases in our examples were simple indefinites interpreted as existential quantifiers. In this chapter, we will turn to quantificational noun phrases and their dynamic semantics.

Recall from chapter 2, that the DRT analysis of quantificational NPs (QNPs) successfully modelled the following aspects: (i) QNPs are distributive;[1] (ii) QNPs introduce their full reference set; (iii) QNPs introduce maximal antecedents that correspond to indefinites in their scope; (iv) QNPs cannot antecede pronouns in their scope, without leading to a bound variable interpretation; and (v) a QNP may introduce dependencies in discourse.

DRT established this success by three theoretical proposals. First of all, QNPs introduce static, distributive quantificational structures (duplex conditions). Second, these structures in turn trigger the introduction of referents identified with abstractions over conditions in the triggering duplex condition. Finally, subsequent quantification over such a referent identified with an abstraction triggers the accessibility of referents involved in the abstraction. This latter principle was introduced only to account for (v) above. Duplex conditions account for (i) and in tandem with abstraction, they explain (iv). Abstraction by itself takes care of (ii) and (iii).

The aim of this chapter is to cover the same aspects of quantification

[1] Recall, however, that this is a simplifying generalisation. See footnote **??** of chapter 2. I leave the data mentioned there for further research.

using just a single proposal: quantificational noun phrases involve a distributive evaluation of their restrictor and scope. The specific semantics of this distributive interpretation will take care of all referential effects.

Let us briefly look ahead and give a preview of the proposal, by abstracting away from the IDPlL framework and the use of states and stacks. Van den Berg's approach to distributivity applied to the running example in (6.1) can roughly be paraphrased as in (6.2).

(6.1)

(a). Three students each wrote a paper.
(b). They each sent it to L&P

(6.2)

(a). Call r a set of three students
Call r' the set of student-paper pairs in the 'write' relation containing a student in r
Every student in r should be in r'.
(b). Call r'' the set of pairs in r' that are also in the relation 'sent-to-L&P'
Every student in r' should be in r''.

In the paraphrases, r' and r'' denote relations or sets of pairs. Alternatively, in the terminology of chapter 5, r' and r'' are states of size two. The set r is a special kind of relation, namely a *unary* relation, or in terms of chapter 5, a set of one-place stacks. In the paraphrases in (6.2), distributivity is modelled by considering the student-paper pairs in the relation instead of considering the set of students and the set of papers separately. The idea of the approach in the current chapter is to develop this treatment of distributivity into an account of distributive quantification. Consider, for instance, parallel to (6.1) and (6.2), the example in (6.3) and its paraphrase in (6.4).

(6.3)

(a). Most students wrote a paper.
(b). Exactly three students sent it to L&P.

(6.4)

(a). Call r the set of salient students
Call r' the set of student-paper pairs in the 'write' relation where the student is in r
r' should contain a majority of the students in r
(b). Call r'' the set of pairs in r' that are also in the 'sent-to-L&P' relation
r'' should contain exactly three of the students in r'.

In these paraphrases, r, r' and r'' correspond to contexts. Notice the following: in (6.4), r' contains both the students that wrote a paper and the

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papers that were written by a student. Moreover, r'' contains the students that sent *their*(!) paper to Linguistics and Philosophy. We can give similar paraphrases if an indefinite is embedded in the restrictor of a quantificational structure.

(6.5) Most students who read a book enjoyed it.

(6.6) Call r the set of student-book pairs in the 'read' relation
Call r' the set of pairs in r that are also in the 'enjoy' relation
r' should contain a majority of the students in r.

The structure of this chapter is as follows. Section 6.1 develops a strategy for the interpretation of quantificational noun phrases; this is inspired by van den Berg-style distributive interpretation, using the framework developed in chapter 5. Next, in section 6.2, we focus on the maxset and the interpretation of the restrictor. This is necessary because of two complications, namely the conditions on the accessibility of the maximal set and the well-known weak/strong distinction in the interpretation of donkey sentences. In section 6.3, we show that this proposal correctly handles entailment patterns involving decreasing quantifiers and pronouns. In 6.4, the findings of chapter 5 and the current chapter are combined in an exposition of the full proposal.

6.1 Quantification and distributivity

The work of van den Berg, discussed in chapter 3, shows us that maximalisation of reference to quantificationally embedded indefinites and dependency phenomena are really two sides of the same coin. Once context keeps track of functional dependencies, it will have to specify all values of the indefinite. That is, a van den Bergian representation of 'the boys who wrote a paper', necessarily also represents 'the papers written by boys', since each boy will be paired with *his* paper. The key to constructing these representations of context was distributivity, which combines updates at the level of atomic entities to a single update involving pluralities.

As we mentioned above, quantificational noun phrases have much in common with this view on distributivity: they are distributive, they access and create dependencies and they trigger maximal reference to embedded indefinites. It therefore seems a good idea to model the dynamics of quantificational noun phrases on the basis of van den Berg-style distributivity.

Recall how the δ operator changed the context. Say we have a state s and say we are interested in the set assigned to slot i. Distributing over this set creates contexts which are formed by appending a state $s\upharpoonright_{i=d}$ to s for a succession of d's in $s[i]$. Each such an extended context is used to interpret the scope of δ and results in an output state which is also an

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extension of s. The combine of all these output extensions is the output for the distribution operation.

In distributive quantification, something similar can be observed. The restrictor sets up a set (the maxset) and the nuclear scope considers the atoms of the set one by one. The only difference is that in distributive quantification it is not always necessary for all the atoms to satisfy the scopal predicate. In fact, the output to distributive quantification is the collection of parts of the restrictor interpretation that comply with the nuclear scope.

What we are interested in, then, is a way of collecting successful updates. Say that we focus, for instance, on a VP 'wrote a paper', which corresponds to the function '$F = \lambda i.\lambda s.(\exists^\star \cdot \text{PAPER}^\star(|s|) \cdot \text{WROTE}^\star(i, |s|))(s)$' and, say, that S is a salient set of students. Consider now the following:

(6.7) $t = \lambda s. \cup \{u \mid \exists d \in S : s^\sqcap u \in F(|s|)(s^\sqcap\{\langle d\rangle\})\}$

Given a state s, this function unifies sets of stacks u which are potential extensions of s with respect to F. In any state s, $t(s)$ is a set of stacks of two positions, representing student-paper pairs such that the student wrote the paper. It collects those extensions of s that form the output of $F(|s|)$ relative to s extended with one of the students in S. In this case, $t(s)$ is the same set of stacks no matter what the input context is like. This is because 'wrote a paper' is completely context independent. Had we chosen a VP like 'sent it to L&P' for F, then different input contexts would have resulted in different states.

We may generalise this way of collecting successful updates as follows:

(6.8) $\sigma := \lambda S_{\langle e,t\rangle}.\lambda P_{\langle \iota,T\rangle}.\lambda s. \cup \{u \mid \exists d \in S : s^\sqcap u \in F(|s|)(s^\sqcap\{\langle d\rangle\})\}$

For instance, in an empty state and a world wherein s_1, s_2 and s_3 wrote the papers p_1, p_2 and p_3 respectively and no other student wrote a paper, $\sigma(I(\text{STUDENT}))(F)$ results in three student-paper pairs such that the student wrote the paper. This is because,

$$
\begin{array}{lcl}
F\left(\begin{array}{c}0\\ \boxed{s_1}\end{array}\right) & = & \begin{array}{cc}0 & 1\\ \boxed{s_1} & \boxed{p_1}\end{array}\\
F\left(\begin{array}{c}0\\ \boxed{s_2}\end{array}\right) & = & \begin{array}{cc}0 & 1\\ \boxed{s_2} & \boxed{p_2}\end{array}\\
F\left(\begin{array}{c}0\\ \boxed{s_3}\end{array}\right) & = & \begin{array}{cc}0 & 1\\ \boxed{s_3} & \boxed{p_3}\end{array}\\
F\left(\begin{array}{c}0\\ \boxed{s_4}\end{array}\right) & = & \emptyset\\
\vdots & & \vdots\\
\text{etc.} & & \text{etc.}
\end{array}
$$

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This operation can already provide the tools for the analysis of simple quantificational expressions. Take for instance 'most students wrote a paper'. Its interpretation will be:

(6.9) $\lambda s.\lambda s'.s' = s^{\sqcap}\sigma(I(\text{STUDENT}))(F)\ \&\ \langle I(\text{STUDENT}), s'[|s|]\rangle \in I(\text{MOST})$

The state s' results from extending s with all the relevant student-paper pairs. Such a state is an output only if the set of students it contains is in the 'most'-relation with the total set of students, $I(\text{STUDENT})$. That is, the set $s'[|s|]$ returns the reference set. Of course, this oversimplifies the role of the restrictor. Most importantly, the restrictor is not simply a set; it may have dynamic effects itself. For instance, the N$'$ 'students who read a book' presents not only a set of students to the scope, but actually student-book pairs. This is because, for each student, the VP has access to the book he or she read. Moreover, restrictors may contain anaphoric material which needs to find an antecedent in the incoming context. We may then use σ for the restrictor interpretation as well. For instance, consider 'students who read a book', interpreted as the $\langle\iota, T\rangle$-function '$G = \lambda i.\lambda s.(\exists^\star \cdot \text{BOOK}^\star(|s|) \cdot \text{READ}^\star(i, |s|))(s)$'. As a restrictor it sets up the state $\sigma(D_e)(G)(s)$, namely the set of student-book pairs such that the student read the book. For instance,

(6.10) $r = \sigma(D_e)(G)(\{\langle\rangle\}) =$

0	1
s_1	b_1
s_2	b_2
s_3	b_3
s_4	b_4

This state can subsequently be used as a base for the interpretation of the scope. Take 'wrote a paper' again and its interpretation as F, above. The VP considers student-book pairs one by one. However, we cannot use σ to this end, since it takes sets, not sets of stacks. We therefore need a means of collecting successful updates which is even more like δ, in that it considers states extended with a sub-state:

(6.11) $\varsigma := \lambda t.\lambda i.\lambda P.\lambda s. \cup\{u \mid \exists d \in t[i] : s^{\sqcap}u \in P(i + |s|)(s^{\sqcap}t\restriction_{i=d})\}$

Let me explain what this says. The function ς takes a state t and an index i and a function of type $\langle\iota, T\rangle$ to return a state modifier. In a state s, it extends s with parts of t that assign a single atomic value d at i. The extensions of s, u, that result from interpreting P with respect to this atomic individual are collected. For instance, for 'DET students who read a book wrote a paper' we can now analyse the scope interpretation as $\varsigma(r)(0)(F)$, which in an empty state results in:

(6.12) $r' = \varsigma(r)(0)(F)(\{\langle\rangle\}) =$

0	1	2
s_1	b_1	p_1
s_2	b_2	p_2
s_3	b_3	p_3

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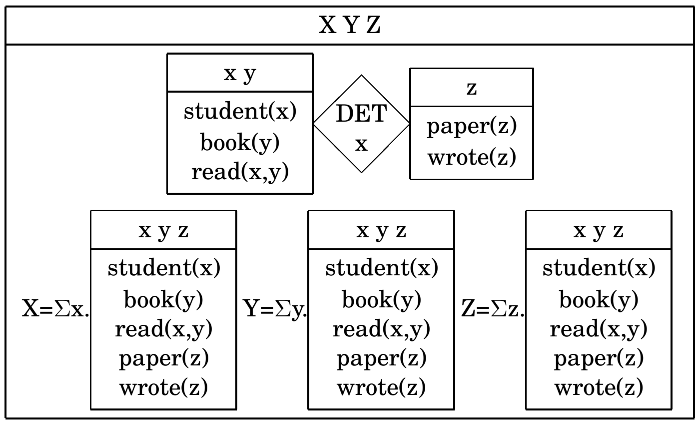
We can now compare the set of students in r at 0 with that in r' at 0. These two sets are supposed to be in accordance with the relation between sets expressed by the determiner. If this is so, r' seems a likely candidate for an output state of the quantificational structure. Not only is $r'[0]$ the full reference set (the set of students that read a book and wrote a paper), r' also has exhaustively collected the values of the indefinites embedded in the quantificational structure. In fact, compare r' with the interpretation of the following DRS in the same situation. A duplex condition corresponding to 'DET students who read a book wrote a paper' triggers three abstraction procedures, as shown in (6.13). In this DRS, X corresponds to $r'[0]$, Y to $r'[1]$ and Z to $r'[2]$.

(6.13)



In the dynamic setup we are pursuing, however, there is no need for a separate procedure for quantification (duplex conditions in DRT) and antecedent formation (abstraction in DRT). In constructing the state r' above, using the state r we already have all the future antecedents we need. The truth-conditions of quantification, moreover, boil down to nothing more than a comparison of r' and r.

Recall, however, that DRT needed to stipulate a third principle governing plural anaphora, which said that if an abstracted set is quantified over, the material used in the abstraction becomes accessible. In our construction of the set of stacks r (the interpretation of the restrictor), merely sets are taken into account, so we cannot expect to be able to predict the effects of this principle as yet.

For instance, in an example like (6.14), the second sentence needs access to the individual papers that were introduced in the first sentence.

(6.14) Every student wrote a paper. Most students sent it to L&P.

The restrictor 'students' in the second sentence should therefore take its set of students from the context, so that the VP 'sent it to L&P' is able to consider student-paper pairs. This means, that instead of 'σ', we should

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use 'ς' to set up the restrictor as well. Say, for instance that the first sentence in (6.14) results in a state with the three student-paper pairs we mentioned above (t_1 in (6.15)). Say now that only the students s_1 and s_2 (which qualify as a majority) sent their paper to L&P. Given an obvious resolution for the object pronoun, the following states are thus involved in the interpretation of (6.14).

(6.15) $t_1 =$

0	1
s_1	p_1
s_2	p_2
s_3	p_3

(6.16) $t_2 = \varsigma(t_1)(0)(\text{STUDENT}^\star)(t_1) = t_1$

(6.17) $t_3 = \varsigma(t_2)(0)(\text{SENT\_TO\_L\&P}^\star(2,3))(t_1) =$

0	1
s_1	p_1
s_2	p_2

The restrictor 'students' uses the input state t_1 as a resource for collecting the salient set of student plus the sets dependent on it. In effect, it returns the input state, so $t_1 = t_2$. The scope 'sent it to L&P' collects those student-paper pairs in t_2 such that the student sent the paper to L&P. The state t_2 is thus reduced to containing only two pairs.

We can now come to a general interpretation scheme for determiners. Say that Q is interpreted in the model as a relation between two sets, then:

(6.18) $$Q_i^\star := \lambda R.\lambda S.\lambda s.\lambda s'.\exists r, r' : s' = s^\sqcap r' \;\&\; \langle r[i], r'[i]\rangle \in I(Q) \;\& \\ r = \varsigma(s)(i)(R)(s) \;\& \\ r' = \varsigma(r)(i)(S)(s)$$

The function $Q_i^\star$ is of a type which is to be expected: $\langle\langle \iota.T\rangle, \langle\langle \iota, T\rangle, T\rangle\rangle$. The index i supplies the so-called context set for quantification (Westerståhl 1984). It makes sure that the domain of quantification is contextually restricted with $s[i]$. Let us for now, for sake of simplicity, assume that determiner functions are magically decorated with such an index. (We will address the issue when we turn to the strong/weak distinction in section 6.2.)

In (6.18), '$r[i]$' corresponds to the maximal set. The state r is formed taking atoms x from $s[i]$ and testing whether they satisfy the restrictor condition R in s extended with $s\restriction_{i=x}$. The reference set is $r'[i]$. It is formed by taking atoms x from $r[i]$ and testing whether they satisfy the scope condition S in s extended with $r\restriction_{i=x}$. Since r', rather than r, is appended to s to yield the output state, only the reference set is introduced in discourse.

Here is an example. The function in (6.19) is the interpretation of 'most students wrote a paper'. It is a function taking a state s and incrementing it with a reference set of students (at slot $|s|+i$) and a set of papers parallel to this reference set.

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(6.19) $\text{MOST}^{\star}_{i}(\text{STUDENTS}^{\star})(\lambda i.\lambda s.((\exists^{\star} \cdot \text{PAPER}^{\star}(|s|) \cdot \text{WROTE}^{\star}(i,|s|))(s)))$

Say we apply this function to some state s. This is what happens: first we take atoms d from $s[i]$ and increment s with $s\lceil_{i=d}$, yielding (say) s'. Then we form the state $(\lambda i.\text{STUDENTS}^{\star}(i))(i+|s|)(s')$. This returns either s' or $\emptyset$, depending on whether or not d is a student. If we collect all such updates into a set r, this state represents the distributive update potential of the restrictor. For the interpretation of the scope we now increment s with the atomic parts of r. These updates result in incrementations of s containing a student in slot $|s|+1$ and they are collected in a state r'. We can now compare how many students there are in r with how many there are in r'. If this comparison is conform the relation expressed by 'most', then we output s incremented with r'.

As another example consider the input context (6.20) to the function in (6.21) and (6.22) respectively.

(6.20) $s =$

0	1
s_1	p_1
s_2	p_2
s_3	p_3
s_4	p_4

(6.21) $\text{MOST}^{\star}_{0}(\text{STUDENT}^{\star})(\lambda i.\text{SENT\_TO\_L\&P}^{\star}(i,3))$

(6.22) $\text{MOST}^{\star}_{0}(\text{STUDENT}^{\star})(\lambda i.\text{SENT\_TO\_L\&P}^{\star}(i,1))$

Whereas (6.21) corresponds to a likely resolution of 'most students sent *it* to L&P', the function in (6.22) is likely to originate from 'most students sent *them* to L&P' (i.e. describing the rather odd situation wherein most students sent all four papers to L&P). Both forms are indexed with 0, that is, the set of students in (6.20) is taken as the context set. In both forms, then, the restrictor expands the input with a pair of a student and a paper in s and checks whether in that expanded state the position $2(=|s|+0)$ is filled by a student. In effect, this returns s again. These same pairs are considered for the nuclear scope. That is, the function in the second argument position of $\text{MOST}^{\star}$ is applied to a context which results from appending $s\lceil_{0=d}$ to s for students d. In such contexts, the position 3 is the paper that paired with the student d in s, while the position 1 is the total set of papers in s. The successful extensions $s\lceil_{0=d}$ are collected in a state r'. In case students in r' are a majority of the students in s, we get an output, namely $s^{\sqcap}r'$.

The procedural nature of this exposition of the semantics of distributive quantification is not for presentation purposes only. It illustrates a key contrast with the interpretation of referential noun phrases. Quantificational NPs introduce a set, distributively present this set as an argument for the VP and then check whether the determiner relation is satisfied. Referential noun phrases, on the other hand, introduce a set, count and

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then interpret the VP with respect to this set. The two strategies can be paraphrased as follows: for QNPs– 'introduce atoms x ; predicate over the atoms ; check whether the successful x's form a witness set' and for RNPs – 'introduce a set X ; check whether X is a witness ; predicate over X'. Notice how these paraphrases explain the differences between RNPs and QNPs mentioned above. In the quantificational case, maximality, distributivity, dependency and the inaccessibility of the reference set within the scope follow directly. (Compare with Szabolcsi 1997 for a statement of similar strategies within DRT terminology. See also Winter 1998.)

There is another contrast between the quantificational and the referential strategy of introducing entities in the discourse. RNPs range over non-empty subsets of the domain of entities, while QNPs include the empty set. This means that all downward monotone NPs will have to be analysed as being quantificational, since they, by definition, include the empty set in their denotation. This is not surprising, however, given that with respect to many properties, downward monotone NPs systematically pair with QNPs. For instance, they are distributive and their refset is accessible only outside of their scope.

A noun phrase like 'no students,' then, is analysed as the $\langle\langle\iota, T\rangle, T\rangle$ function '$\lambda P.\textsc{no}^{\star}_{i}(\textsc{student}^{\star})(P)$'. Notice that this function for some verb phrase P and some state s either returns the empty set or returns s. This is because the relation NO can only be satisfied when there is no extension of s that satisfies the conditions in the nuclear scope. In other words, a sentence '[[No N′] VP]' is (correctly) analysed as a test.

Let us evaluate the merits of the definition in (6.18). We already established that ς constructs states in such a way that both the refset and sets depending on the refset are exhaustively represented in context. Moreover, ς assures a purely distributive interpretation by considering sets atom by atom only. Note, moreover, that the full refset is only represented in context once the interpretation of the quantificational structure is completed.

In sum, the above proposal reaches results comparable to those of DRT, using a single interpretational strategy of determiners. Let us, now, turn to other aspects of quantificational structures. We begin with discussing the maximal set.

6.2 The maxset

We have remained silent about the accessibility of the maximal set. The interpretation of a quantificational noun phrase as given above predicts the following: the maximal set is at no point accessible. The reason is that at no point in the evaluation is the full set of successful updates with the restrictor (systematically called 'r' above) taken as an input context. The scope only considers atoms from this set and a successful quantificational structure only increments the input state with states due to the scope.

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This begs the question as to how the maxset can sometimes function as an antecedent. However, given that we have so far assumed that restrictors take their value from a salient set in the context, it follows that the (potential) accessibility of maxset is due to the fact that it already occurs in context. In other words, its existence was presupposed.

We will now consider two details of how restrictions are interpreted which deserve more attention. Both deal with a (different) weak/strong distinction in quantification. The first concerns the weak and strong readings of donkey sentences. The second addresses the weak/strong distinction in the sense of intersective/non-intersective quantifiers.

6.2.1 Weak versus strong readings

The interpretation of a restrictor collects every possible extension of the current information state that satisfies the restrictor clause. However, in the case of complex restrictors containing indefinites, there is no straightforward way of choosing how to collect these extensions. For instance, the N′ 'boys who wrote a paper' extends the input state with relevant boy-paper pairs. However, when one of the boys wrote more than one paper, there are two options: either we collect all the pairs, entering some boys more than once into the extension (call it the 'A'-option), or we allow extensions which do not exhaustively collect all pairs, as long as all boys that wrote a paper are in the extension (call it 'B'). The choice between these two ways of building up restrictions is not straightforward.

For example, in a situation with three boys writing a paper, wherein b_1 wrote p_1 and b_2 wrote p_2 and b_3 wrote p_3 and p_4, (6.23a) gives the A-option for extending the context, while (6.23b) and (6.23c) give two possible B-style extensions.

(6.23) a.

0	1
b_1	p_1
b_2	p_2
b_3	p_3
b_3	p_4

b.

0	1
b_1	p_1
b_2	p_2
b_3	p_3

c.

0	1
b_1	p_1
b_2	p_2
b_3	p_4

The scope is interpreted by extending the input context with parts of one of the above states. For instance, for the A-option, in a state s a nuclear scope 'sent it to L&P' will be interpreted by considering the state s, extended with (6.23a)$\upharpoonright_{0=b_i}$ for i ranging over 1, 2, and 3. With respect to the third boy, this means the scope is interpreted as follows (assuming 'it' to be resolved to be dependent on the papers):

(6.24) $$\text{SENT\_TO\_L\&P}(|s|, |s|+1)\left(s^{\sqcap}\begin{array}{|c|c|} \multicolumn{1}{c}{0} & \multicolumn{1}{c}{1} \\ \hline b_3 & p_3 \\ \hline b_3 & p_4 \\ \hline \end{array}\right)$$

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In other words, according to the A-option, in order for the sentence 'D boys who wrote a paper sent it to L&P' to be true, it should hold that D boys sent *all* the papers they wrote to L&P.

In the 'A' set-up, pronominal reference to the indefinite inside the restrictor clause will be interpreted as exhaustive. That is, for each boy it will retrieve all the papers he wrote. This is not the case for the 'B' option, where the nuclear scope may be false for one of the papers written by a particular boy as long as there exists another one he did sent to L&P.

All this is closely connected to the well-known distinction between weak (or existential) versus strong (or universal) readings of quantificational sentences. For instance, while (6.25) is preferably interpreted weakly as being true when every guest owning one or more credit cards uses one of his cards to pay the bill, (6.26) seems to lead to a stronger reading in which every farmer is required to beat all the donkeys he owns. The weak paraphrases (indicated as ∃) and the strong paraphrases (indicated as ∀) are as in (6.27) and (6.28), after Kanazawa 1994.

(6.25) Every guest who owned a credit card, used it to pay the bill.

(6.26) Every farmer who owns a donkey, beats it.

(6.27) Q farmer who owns a donkey, beats a donkey he owns. (∃)

(6.28) Q farmer who owns a donkey, beats every donkey he owns. (∀)

In absence of conflicting information, it seems that 'every' triggers a strong reading. The knowledge that it takes only one credit card to pay a bill, weakens the reading for (6.25). Not all determiners show similar preference patterns for weak and strong readings however. For instance, (6.29) is only verified when no farmer beats *any* donkey he owns, not just some of them. In fact, the strong reading seems impossible to obtain.

(6.29) No farmer who owns a donkey, beats it.
#*"No farmer who owns a donkey beats every donkey he owns"* (∀)
"No farmer who owns a donkey beats a donkey he owns" (∃)

Some downward entailing environments, however, do not seem to require a similarly weak reading. Kanazawa (1994) points out that an example like (6.30) seems to favor a strong reading. Krifka (1996), however, shows that this might be due to the fact that 'not every' is not a determiner, since the semantically similar "less than 100%" pairs with other determiners which are downward entailing in their second argument and requires an existential interpretation.

(6.30) Not every farmer who owns a donkey beats it.
"Not every farmer who owns a donkey beats every donkey he owns" (∀)
?*"Not every farmer who owns a donkey beats a donkey he owns"* (∃)

From: Nouwen, R. (to appear) "Plural pronominal anaphora in context: dynamic aspects of quantification." PhD-thesis, UiL-OTS, Universiteit Utrecht. LOT-dissertatiereeks.

(6.31) Less than 100% of the farmers who own a donkey beat it.
?*"Less than 100% of the farmers who own a donkey beat every donkey they own"* (∀)
"Less than 100% of the farmers who own a donkey beat a donkey they own" (∃)

In our current approach, the distinction between weak and strong comes out rather different. What we have called the 'A' option, where all values are bundled together and consequently pronouns in the scope have to exhaustively refer to an indefinite in the restrictor clause, will provide the strong reading for upward entailing quantifiers, but the weak reading for downward monotone ones. Consider, for instance (6.32), below, and its paraphrase in (6.33).

(6.32) No farmer who owns a donkey beats it.

(6.33) Call r the set of farmer-donkey pairs in the own relation.
Call r' the set of pairs in r that are also in the 'beat' relation.
r' should be the empty set.

This is in accordance with the ∃-paraphrase in (6.29). The weakness of (6.33) is due to the fact that the quantificational relation (and hence its monotonicity properties) only has its effect after processing the scope. Notice that the approach to the interpretation of quantificational noun phrases set out above represents the A-approach. For the 'B' option, upward quantifiers are interpreted as being weak, while downward ones receive a universal reading. It seems then that although all determiners prefer the 'A' option, sometimes a shift is made toward generating multiple environments.[2]

Let me briefly turn to a related issue. One would expect that the weak/strong distinction at the sentence level also occurs on discourse level. That is, it would not be surprising if the discourse in (6.34) were ambiguous between a reading wherein all papers written by the students were not very good and one wherein any set of papers, each of them written by one of the boys, were not good.

(6.34) Every student wrote a paper. They weren't very good.

Although it is difficult to get clear judgments, it seems that strong readings are by far the more preferred ones. In fact, it is questionable whether weak readings really exist. Intuitively, the second sentence in (6.34) is false when one of the three students wrote one bad and one good paper. (See van der Does 1993 for a similar speculation.)

Nevertheless, an example like (6.35) might complicate things.

[2]See, especially, Kanazawa 1994 and Geurts 1999 for two interesting discussions of factors involved in the availability of and preferences for certain readings.

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(6.35) That day, all the guests wanted to leave the hotel. Unfortunately, the banks were closed. Luckily, every guest owned a credit card. They used them to pay their bills.

As in (6.25), (6.35) does not require a guest owning more than one credit card to use all his cards to pay the bill. One could argue, however, that the strong reading is not that bad for this example. It simply says that every guest used his credit cards to pay the bill. That is, he payed by credit card and it does not matter which of his credit cards he used. In other words, the guest uses his credit resources to pay the bill.

The proposal in this chapter predicts that only strong readings exist in discourse. This is again due to the way the ς-operation collects successful updates. It does so exhaustively, so following 'every guest owned a credit card', the refset is paired with all the cards owned by the guests. Although my remarks above seem to strongly suggest that this is a welcome prediction, I leave it an open question as to whether or not apparent counter-examples like (6.35) can be accounted for by other means.

6.2.2 Parasitic domains

Let us deal with the index-decoration of determiners. We suggested that a structure $Q(A)(B)$ is to be translated into the function $Q_i^\star(A^\star)(B^\star)$. Here, the purpose of the index i was to supply the contextual restriction of the domain of quantification (following, again, Westerståhl (1984)).

Of course, such an index decoration is not compositionally given. In effect, the quantifier is anaphoric. Or, to be more precise, in cases where the quantifier is strong and therefore presupposes its maxset, this presupposition may be anaphorically resolved. So instead of the function $Q_i^\star(A^\star)(B^\star)$, quantificational structures should correspond to underspecified representations: $Q_0(A)(B)$.

Note however that weak quantifiers (that is, intersective ones) do not need a contextually supplied context set. In existential-there constructions, for instance, clearly no domain restriction is needed.

(6.36) There are four boys in the garden.

This example is intelligible even if no set of salient boys exist in context. In fact, it simply reports the existence of such a salient set. Another example is the well-known observation that the object of verbs like 'have' may only be weak (see, e.g., de Hoop 1992). In (6.37), then, no salient set of windows is assumed. The obligatorily strong determiner 'most' is banned from such positions, since it requires such an antecedent.

(6.37) This house has many windows.

(6.38) *This house has most windows.

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Consequently, weak quantifiers are in need of a different interpretation, one which has no anaphoric properties. The following provides a straightforward solution. Instead of taking a set from a slot in the input context, the weak quantifier takes its values from a state $\{\langle d\rangle | d \in D_e\}$, the state containing only the universe.

(6.39) $Q^\star_{\mathsf{W}} := \lambda R.\lambda S.\lambda s.\lambda s'.\, \exists r, r' : s' = s^\sqcap r' \;\&\; \langle r[i], r'[i]\rangle \in I(Q) \;\&$
$r = \varsigma(\{\langle d\rangle | d \in D_e\})(0)(R)(s) \;\&$
$r' = \varsigma(r)(i)(S)(s)$

According to this definition, in the interpretation of a weak quantifier, the maximal set (given by r) serves only as a resource for the interpretation of the scope. It is not supposed to be given by the context and consequently is never available for anaphora.

6.3 Entailment and emptiness

Recall from chapter 2 and 3 that pronominal reference to the refset of a downward entailing quantifier demonstrates the non-emptiness of that set, whereas this is not guaranteed by the antecedent quantification by itself.

(6.40) Few senators admire Kennedy.
$\not\Rightarrow$ Some senator admires Kennedy.

(6.41) Few senators admire Kennedy. They are very junior.
$\Rightarrow$ Some senator admires Kennedy

Let us, for the sake of simplicity, assume that 'few' is interpreted as weak, here. (Although nothing hinges on this). That is, 'Few senators admire Kennedy' is interpreted as the function in ('6.42')

(6.42) $\text{FEW}^\star_{\mathsf{W}}(\text{SENATOR}^\star)(\text{ADMIRE\_K}^\star)$

Consider now a world wherein none of the senators admire Kennedy. Then, call r the set of one-place stacks comprising the set of senators. Then r' is the set of states $r\restriction_{0=d}$ such that d admires Kennedy. Since there is no such d, it follows that $r' = \emptyset$. Clearly now, r and r' satisfy the 'few'-relation, so the output to (6.42) in this world for any s is s again. In other words, (6.42) is true in this world, but given that no senator admiring Kennedy was found, the input state was not incremented. (Or, in fact, the input state has been incremented with the empty set.) This shows that (6.40), in the form (6.43) below, holds.

(6.43) $\text{FEW}^\star_{\mathsf{W}}(\text{SENATOR}^\star)(\text{ADMIRE\_K}^\star) \not\models$
$\lambda s.\exists^\star \cdot \text{SENATOR}^\star(|s|) \cdot \text{ADMIRE\_K}^\star(|s|)(s)$

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Turning now to the influence of the pronoun, the sentence 'they are very junior' is represented as 'VERY_JUNIOR($_0$)'. In a world as described above, i.e. one wherein no senator admires Kennedy, and following (6.42) in context s, the resolutions of this representation are those wherein an index $i < |s|$ is substituted for $_0$. That is, there simply is no possible resolution to the refset. Consequently, when the pronoun *is* resolved to the refset of (6.42), we know that we are in a world wherein some senators admire Kennedy. The entailment in (6.41) follows.

More formally,

(6.44) $\lambda s.(\text{FEW}^{\star}_{\mathsf{W}}(\text{SENATOR}^{\star})(\text{ADMIRE\_K}^{\star}) \cdot \text{VERY\_JUNIOR}^{\star}(|s|))(s) \models$
$\lambda s.\exists^{\star} \cdot \text{SENATOR}^{\star}(|s|) \cdot \text{ADMIRE\_K}^{\star}(|s|)(s)$

The proof is easy. There is only a single possible output state resulting from combining the function in front of '$\models$' with some state s. In models containing few, but some, Kennedy admiring senators, this function returns the input state s incremented with the set of these admirers. In any other model, the function is undefined since there is no slot $|s|$ at the point where $\text{VERY\_JUNIOR}^{\star}(|s|)$ is encountered. So, in those worlds, this function would have never resulted from resolution. It follows then that all contexts resulting from applying this function to some input state yield an output after applying $\lambda s.\exists^{\star} \cdot \text{SENATOR}^{\star}(|s|) \cdot \text{ADMIRE\_K}^{\star}(|s|)(s)$ to it.

Given these observations, note, however, the following: the text '*Few senators admire Kennedy. They are very junior.*' does not possibly-entail '*Some senators admire Kennedy*' (put more formally in (6.45). This is because in worlds without Kennedy-admiring senators, there will be no resolution to the refset and 'they' will be forced to take a completely different antecedent. In sum, only the resolved case, where 'they' picks up the reference set, allows us to infer the non-emptiness of this set.

(6.45) $\text{FEW}_{\mathsf{W}}(\text{SENATOR})(\text{ADMIRE\_K}) \cdot \text{VERY\_JUNIOR}(0) \not\mid\!\!\leadsto$
$\exists \cdot \text{SENATOR}(0) \cdot \text{ADMIRE\_K}(0)$

The following two patterns are left to the reader to check.

(6.46) Most students are bachelors $\mid\!\!\leadsto$ They are unmarried

(6.47) Few students are bachelors $\not\mid\!\!\leadsto$ They are unmarried

We have shown here that the proposal predicts the presuppositional nature of a pronoun, by the fact that empty values are not stored in context. Pronoun resolution to the refset is therefore only possible in worlds wherein this set is not empty.

6.4 The full proposal

In this section, we will give an overview of the full account of quantification and anaphora. Some definitions will be repeated from earlier chapters.

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In this section, we will collect all manipulations we have defined on our notion of context, namely that of a set of stacks. Our ontology is as follows. D_e is the domain of entities of type e. The set of indices of type ι is $\mathbb{N}$.

Definition 6.1 States

$$s :: \langle\langle\iota, e\rangle, t\rangle \text{ is a state } \Leftrightarrow \exists n \in \mathbb{N} : \ s \subseteq \{\langle d_1, \ldots, d_n\rangle \mid d_1, \ldots, d_n \in D_e\}$$

$$\begin{array}{rcl} s[i] & := & \{c(i) \mid c \in s \ \& \ c(i) \neq\uparrow\} \\ |s| & := & \iota n.\forall c \in s : |c| = n \\ s\restriction_{i=d} & := & \left\{\begin{array}{ll} \{c \in s \mid c(i) = d\} & i < |s| \\ \uparrow & \text{otherwise} \end{array}\right. \end{array}$$

□

This defines the states and the operations of states we have been using in chapter 5. As we did there, we abbreviate the type $\langle\langle\langle\iota, e\rangle, t\rangle, \langle\langle\iota, e\rangle, t\rangle\rangle$ as T. Let $\mathcal{Q}$ be a set of determiner symbols and $\mathcal{P}$ be a set of predicate symbols. We consider a model $M = \langle D, I\rangle$ which is such that $I(R)$ returns an element in $D_e \times \ldots \times D_e$ for any R in $\mathcal{Q} \cup \mathcal{P}$. We define the following collection of functions:

Definition 6.2 State manipulation

$$\begin{array}{rcll} \varsigma & := & \lambda t.\lambda i.\lambda P.\lambda s. \cup \{u \mid \exists d \in t[i] : s^\sqcap u \in P(i + |s|)(s^\sqcap t\restriction_{i=d})\} & \\ Q_i^\star & := & \lambda R.\lambda S.\lambda s.\lambda s'. \exists r, r' : s' = s^\sqcap r' \ \& \ \langle r[i], r'[i]\rangle \in I(Q) \ \& & \text{for } Q \in \mathcal{Q} \\ & & \quad r = \varsigma(s)(i)(R)(s) \ \& & \\ & & \quad r' = \varsigma(r)(i)(S)(s) & \\ Q_{\mathsf{W}}^\star & := & \lambda R.\lambda S.\lambda s.\lambda s'. \exists r, r' : s' = s^\sqcap r' \ \& \ \langle r[i], r'[i]\rangle \in I(Q) \ \& & \text{for } Q \in \mathcal{Q} \\ & & \quad r = \varsigma(\{\langle d\rangle | d \in D_e\})(0)(R)(s) \ \& & \\ & & \quad r' = \varsigma(r)(i)(S)(s) & \\ P^\star & := & \lambda i_1, \ldots, i_n.\lambda s.\lambda s'. s = s' \ \& \ \langle s[i_1], \ldots, s[i_n]\rangle \in I(P) & \text{for } P \in \mathcal{P} \\ \exists^\star & := & \lambda s.\lambda s'.\exists X \in \wp^+(D_e) : s' = \{c^\wedge d | c \in s \ \& \ d \in X\} & \\ \neg^\star & := & \lambda s.\lambda s'. s = s' \ \& \ \varphi s = \emptyset & \\ \delta^\star & := & \lambda P.\lambda i.\lambda s.\lambda s'. \, s[i] = s'[i] \ \& & \\ & & \quad \forall x \in s[i] : \ s^\sqcap s'\restriction_{i=x} \in P(i{+}|s|)(s^\sqcap s\restriction_{i=x}) & \end{array}$$

$$(\varphi \cdot \psi) \ := \ \lambda s. \bigcup\{\psi s' | s' \in \varphi s\}$$

□

Some of these functions are actually partial. Here are the definedness conditions:

Definition 6.3 Definedness

$$\begin{array}{rcccl} Q_i^\star(R)(S)(s) & = & \downarrow & \Leftrightarrow & i < |s| \ \& \ R(i)(s) =\downarrow \ \& \ S(i)(s) =\downarrow \\ Q_{\mathsf{W}}^\star(R)(S)(s) & = & \downarrow & \Leftrightarrow & i < |s| \ \& \ R(i)(s) =\downarrow \ \& \ S(i)(s) =\downarrow \\ P^\star(i_1)\ldots(i_n)(s) & = & \downarrow & \Leftrightarrow & \forall 1 \leq j \leq n : i_j < |s| \\ \delta^\star(P)(i)(s) & = & \downarrow & \Leftrightarrow & i < |s| \end{array}$$

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□

Truth, validity and entailment are defined on functions of type T:

Definition 6.4 Truth and entailment. For $\varphi, \psi :: T$:

$$\begin{array}{rcll} \models_s \varphi & \Leftrightarrow & \varphi(s) \neq \emptyset & \text{(truth)} \\ \models \varphi & \Leftrightarrow & \forall s : \varphi(s) = \downarrow \rightarrow \varphi(s) \neq \emptyset & \text{(validity)} \\ \varphi \models \psi & \Leftrightarrow & \forall s, s' : s' \in \varphi(s) \rightarrow \psi(s') \neq \emptyset & \text{(entailment)} \end{array}$$

□

The representation language L' for context manipulation is extended to include the atomic formulae Q, for $Q \in \mathcal{Q}$. In L', such a form Q is of type T. Similarly, the underspecification language L is extended to include the same atomic forms. (The details for these languages can be found in section **??**.) Resolution and possible entailment are as before.

Figure 6.1 gives a fragment. It only derives sentence meanings, namely underspecified representation in the language L of type T. Interpretation of multi-sentence discourses need a mediating resolution mechanism. Disambiguated sentence meanings, i.e. state transitions, are composed using '$\cdot$'.

6.5 Conclusion

In this final chapter, we argued that three principles used in the DRT approach to distributive quantification and anaphora can, in the dynamic framework, be replaced by a single interpretative strategy for quantificational noun phrases. By further making a distinction between strong quantifiers, which are anaphoric, and weak quantifiers, which can be interpreted without antecedent, we predict that the former, but not the latter group of NPs license subsequent reference to the maximal set. Finally, since in the proposal quantificational noun phrases increment the input context with the successful output states of the interpretation of their scope, there is no guarantee that a downward entailing NP increments the context at all. Consequently, context will never store an empty value. This accounts for why pronominal reference to the reference set presupposes the non-emptiness of the antecedent.

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S	::=	NP VP	X	::=	$(X_1 X_2)$
NP	::=	Det CN	X	::=	$(X_1 X_2)$
VP	::=	IV	X	::=	X_1
VP	::=	TV NP	X	::=	$\lambda u.(X_2(\lambda v.(X_1 v)u))$
VP	::=	MOD VP	X	::=	$(X_1 X_2)$
NP	::=	they	X	::=	$\lambda P.(P(_0))$
NP	::=	it	X	::=	$\lambda P.(1(_0) \bullet P(_0))$
CN	::=	man	X	::=	MAN
CN	::=	woman	X	::=	WOMAN
TV	::=	sent to L&P	X	::=	SENT_TO_L&P
MOD	::=	each	X	::=	$\lambda P.\lambda i.(\delta(P)(i))$
DET	::=	a	X	::=	$\lambda P.\lambda Q.(\exists \bullet 1(0) \bullet P(0) \bullet Q(0))$
DET	::=	three	X	::=	$\lambda P.\lambda Q.(\exists \bullet 3(0) \bullet P(0) \bullet Q(0))$
DET	::=	many	X	::=	$\lambda P.\lambda Q.(\text{MANY}_{\mathsf{W}}(P(0))(Q(0)))$
DET	::=	few	X	::=	$\lambda P.\lambda Q.(\text{FEW}_{\mathsf{W}}(P(0))(Q(0)))$
DET	::=	at_least_three	X	::=	$\lambda P.\lambda Q.(>3_{\mathsf{W}}(P(0))(Q(0)))$
DET	::=	at_most_three	X	::=	$\lambda P.\lambda Q.(<3_{\mathsf{W}}(P(0))(Q(0)))$
DET	::=	many	X	::=	$\lambda P.\lambda Q.(\text{MANY}_0(P(_0))(Q(_0)))$
DET	::=	few	X	::=	$\lambda P.\lambda Q.(\text{FEW}_0(P(_0))(Q(_0)))$
DET	::=	at_least_three	X	::=	$\lambda P.\lambda Q.(>3_0(P(_0))(Q(_0)))$
DET	::=	at_most_three	X	::=	$\lambda P.\lambda Q.(<3_0(P(_0))(Q(_0)))$
DET	::=	most	X	::=	$\lambda P.\lambda Q.(\text{MOST}_0(P(_0))(Q(_0)))$
DET	::=	less_than_half	X	::=	$\lambda P.\lambda Q.(<.5_0(P(_0))(Q(_0)))$

Figure 6.1: Fragment

From: Nouwen, R. (to appear) "Plural pronominal anaphora in context: dynamic aspects of quantification." PhD-thesis, UiL-OTS, Universiteit Utrecht. LOT-dissertatiereeks.

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